

Laurent Series and z-Transform

- Geometric Series

Permutations C

20251030 Thr

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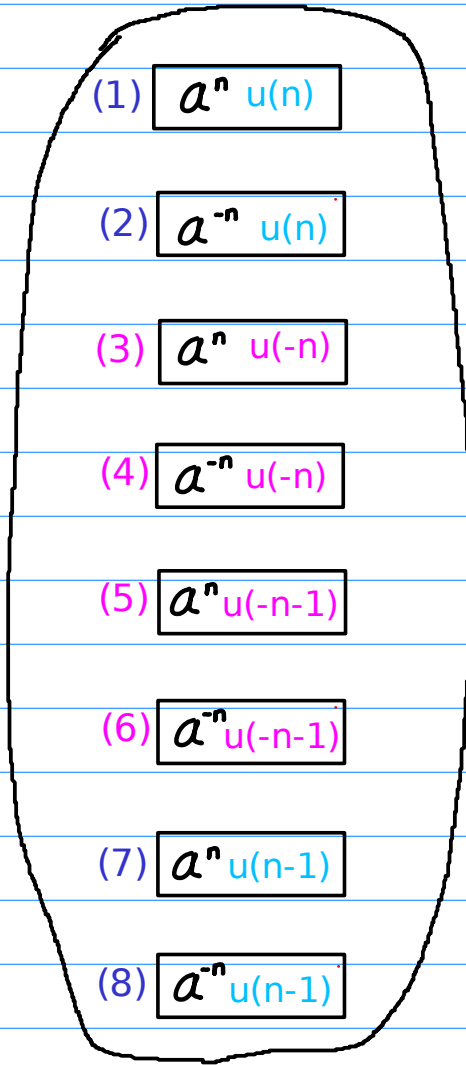
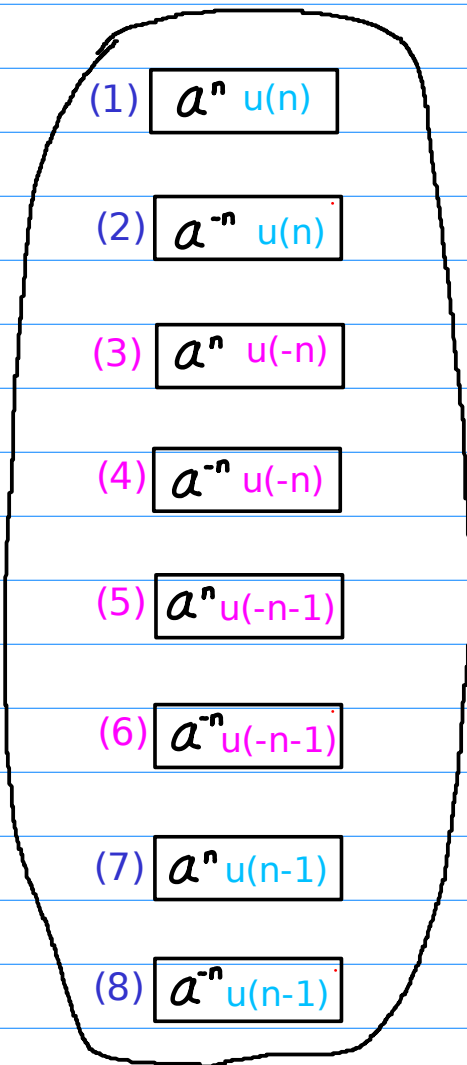
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Intra-Permutations over (1) ~ (8)

Unshifted
Sequence $\times$



Unshifted
Sequence $\times$

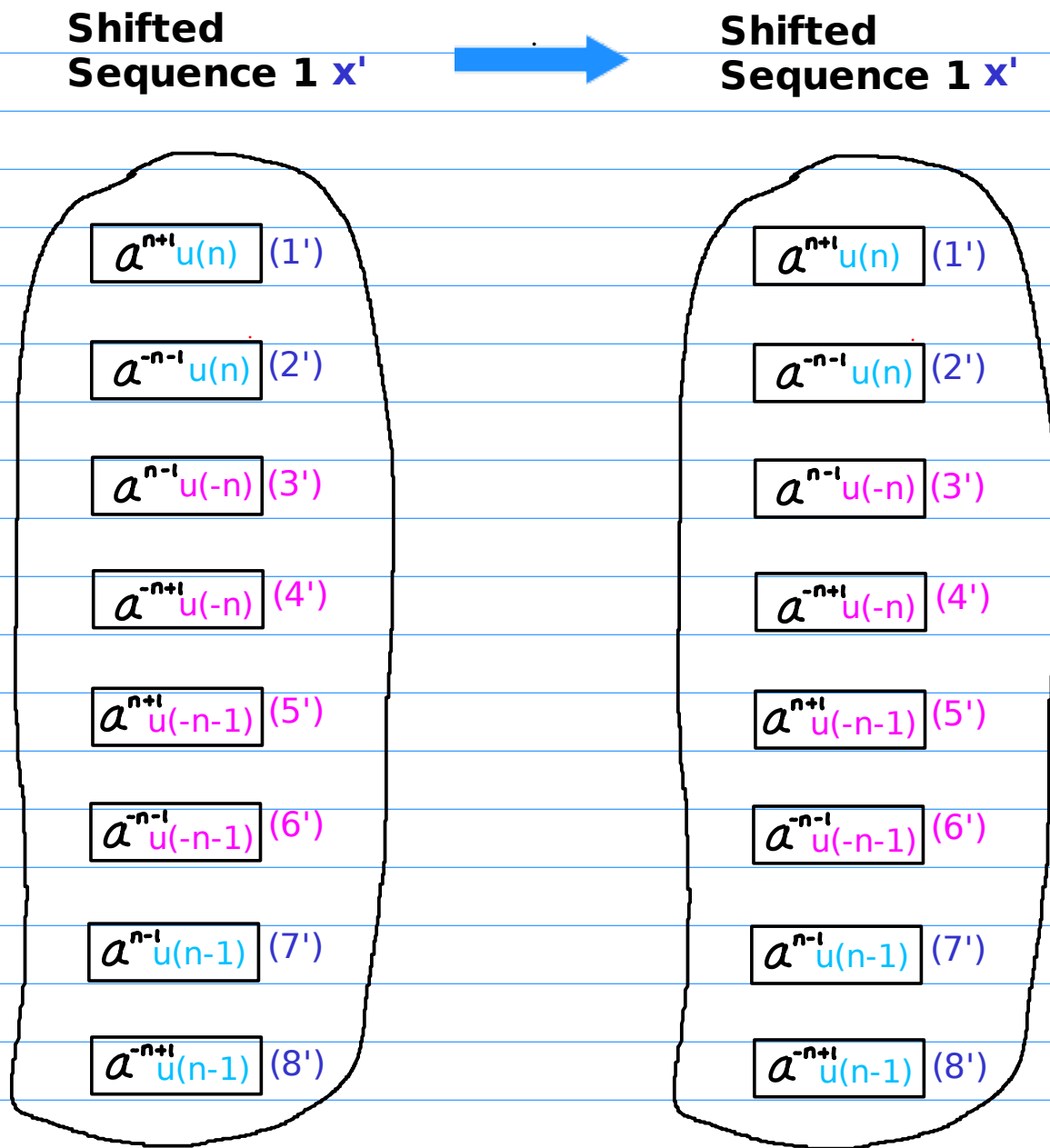


Inter-permutations over unshifted sequence and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence

Intra-Permutations over (1') ~ (8')



Inter-permutations over unshifted sequence and shifted sequence

Intra-permutations over unshifted sequence

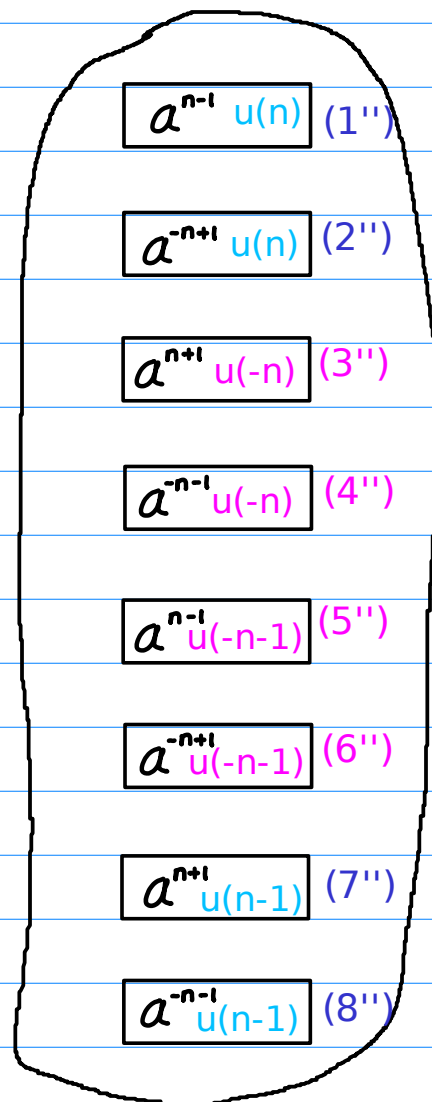
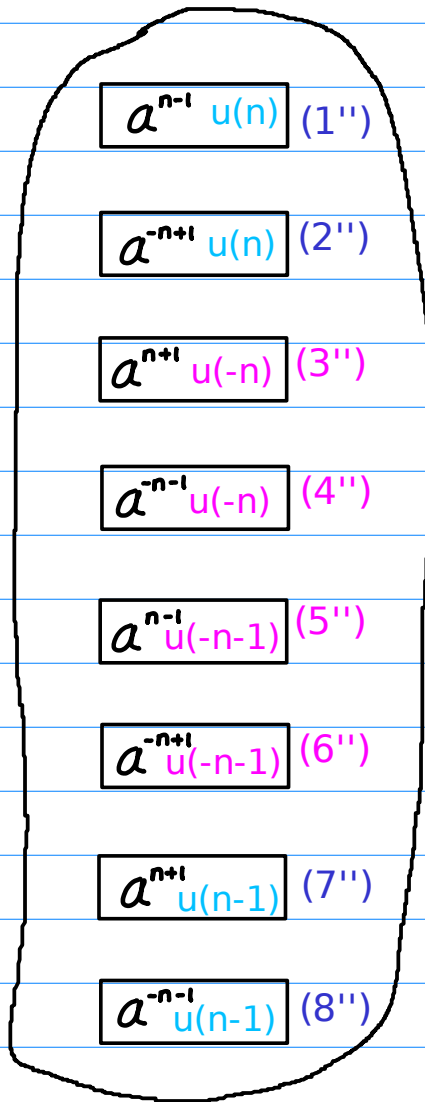
Intra-permutations over shifted sequence

Intra-Permutations over (1') ~ (8')

Shifted
Sequence 1 x''



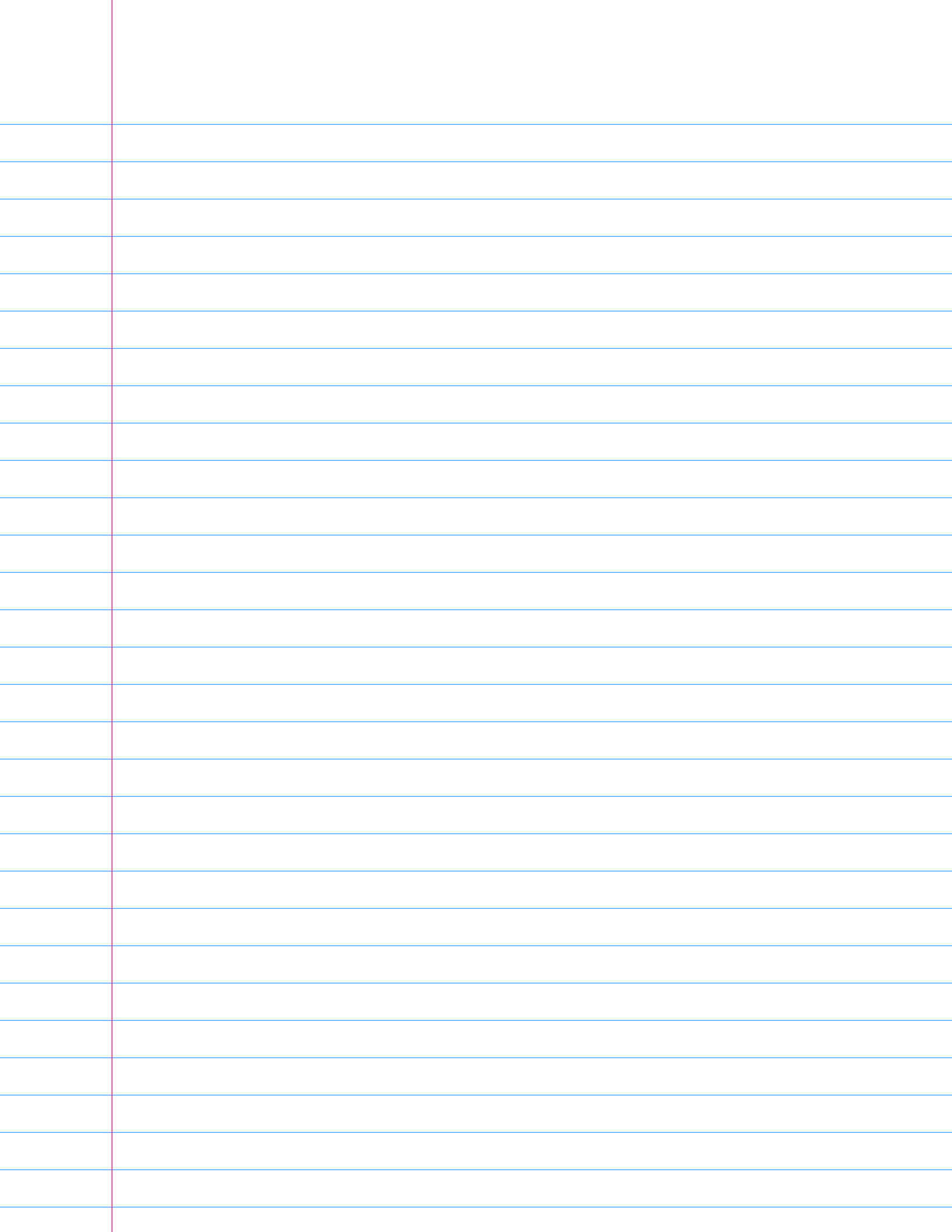
Shifted
Sequence 1 x''

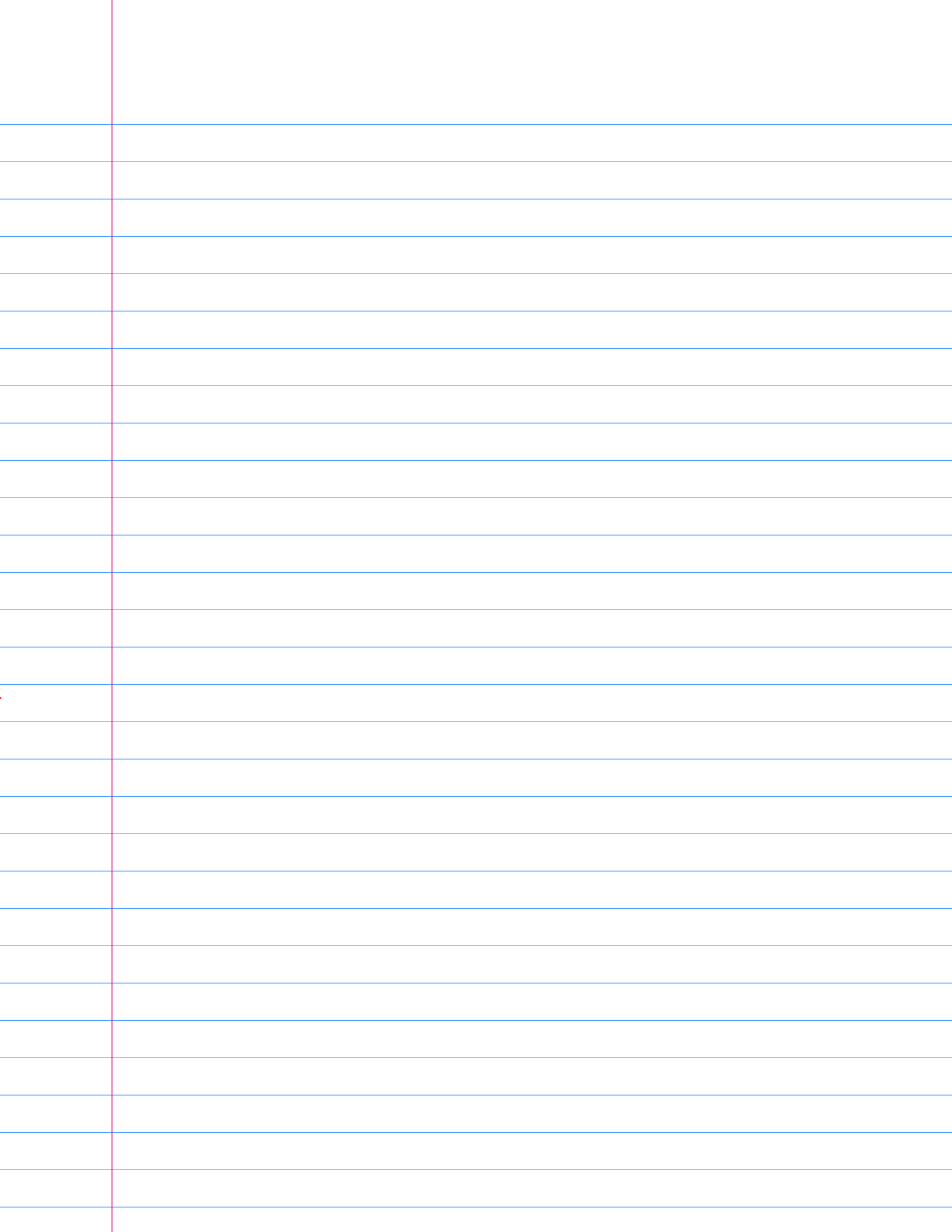


Inter-permutations over unshifted sequence and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence





permutation over (1) ~ (8) (x)  (x)

A. Flipping	(1) - (4)	(5) - (8)
Base Inverting	(2) - (3)	(6) - (7)
Range Flipping	(3) - (2)	(7) - (6)
	(4) - (1)	(8) - (5)
B. <u>Range Shifting</u>	(1) - (7)	(5) - (3)
Range Flipping	(2) - (8)	(6) - (4)
Range Complementing	(3) - (5)	(7) - (1)
	(4) - (6)	(8) - (2)
C. Complementary Inverting	(1) - (6)	(5) - (2)
Base Inverting	(6) - (1)	(6) - (1)
Range Complementing	(2) - (5)	(7) - (4)
	(5) - (2)	(8) - (3)

permutation over (1') ~ (8') (x')  (x')

D. Flipping2	(1') - (4')	(5') - (8')
Base Inverting	(2') - (3')	(6') - (7')
<u>Shifted Range Flipping</u>	(3') - (2')	(7') - (6')
	(4') - (1')	(8') - (5')
E. Shifting2	(1') - (7')	(5') - (3')
Shifted Range Flipping	(2') - (8')	(6') - (4')
Range Complementing	(3') - (5')	(7') - (1')
	(4') - (6')	(8') - (2')
F. Complementary Inverting	(1') - (6')	(5') - (2')
Base Inverting	(6') - (1')	(6') - (1')
Range Complementing	(2') - (5')	(7') - (4')
	(5') - (2')	(8') - (3')

Shifted Range Flipping = Exponent Shifting2 + Range Flipping

Shifting2 = Shifted Range Flipping + Range Complementing
= Exponent Shifting2 + Range Flipping + Range Complementing
= Exponent Shifting2 + Range (Flipping+Complementing)
= Exponent Shifting2 + Range Shifting

permutation over (1'') ~ (8'') (x'') → (x'')

D. Flipping2

Base Inverting

Shifted Range Flipping

(1'') - (4'')	(5'') - (8'')
(2'') - (3'')	(6'') - (7'')
(3'') - (2'')	(7'') - (6'')
(4'') - (1'')	(8'') - (5'')

E. Shifting2

Shifted Range Flipping

Range Complementing

(1'') - (7'')	(5'') - (3'')
(2'') - (8'')	(6'') - (4'')
(3'') - (5'')	(7'') - (1'')
(4'') - (6'')	(8'') - (2'')

F. Complementary Inverting

Base Inverting

Range Complementing

(1'') - (6'')	(5'') - (2'')
(6'') - (1'')	(6'') - (1'')
(2'') - (5'')	(7'') - (4'')
(5'') - (2'')	(8'') - (3'')

Permutation over (1) ~ (8)

Permutations	A	B	C
Base Inverting	X		X
Range Flipping	X	X	
Range Complementing		X	X

Permutation over (1') ~ (8')

Permutations	D	E	F
Base Inverting	X		X
Shifted Range Flipping	X	X	
Range Complementing		X	X

Shifted Range Flipping = Exponent Shifting₂ + Range Flipping

Shifting₂ = Shifted Range Flipping + Range Complementing
= Exponent Shifting₂ + Range Flipping + Range Complementing
= Exponent Shifting₂ + Range (Flipping + Complementing)
= Exponent Shifting₂ + Range Shifting

Over (1) ~ (8)

Base Inverting

$$a^{\textcolor{blue}{n}} \longleftrightarrow a^{-\textcolor{blue}{n}}$$

Range Flipping

$$R(\textcolor{blue}{n}) \longleftrightarrow R(-\textcolor{blue}{n})$$

Range Complementing

$$R(\textcolor{blue}{n}) \longleftrightarrow \overline{R(\textcolor{blue}{n})}$$

Over (1') ~ (8')

Base Inverting

$$a^{\textcolor{blue}{n}} \longleftrightarrow a^{-\textcolor{blue}{n}}$$

Shifted Range Flipping

$$a^{\textcolor{blue}{n}} R(\textcolor{blue}{n}) \longleftrightarrow a^{\textcolor{red}{sh}2(\textcolor{blue}{n})} R(-\textcolor{blue}{n})$$

Range Complementing

$$R(\textcolor{blue}{n}) \longleftrightarrow \overline{R(\textcolor{blue}{n})}$$

A.I Flipping

**Base Inverting
Range Flipping**

$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-n} R(-n)$$

D.I Flipping2

**Base Inverting
Shifted Range Flipping**

$$a^n \longleftrightarrow a^{-n}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-sh2(n)} R(-n)$$

B.I Range Shifting

**Range Flipping
Range Complementing**

$$R(n) \longleftrightarrow R(-n)$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$R(n) \longleftrightarrow \overline{R(-n)}$$

E.I Shifting2

**Shifted Range Flipping
Range Complementing**

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

C.I Complementary Inverting

**Base Inverting
Range Complementing**

$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

F.I Complementary Inverting

**Base Inverting
Range Complementing**

$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

G.I Flipping2

**Base Inverting
Shifted Range Flipping**

$$\begin{aligned}a^n &\longleftrightarrow a^{-n} \\ a^n R(n) &\longleftrightarrow a^{sh2(n)} R(-n) \\ a^n R(n) &\longleftrightarrow a^{-sh2(n)} R(-n)\end{aligned}$$

H.I Shifting2

**Shifted Range Flipping
Range Complementing**

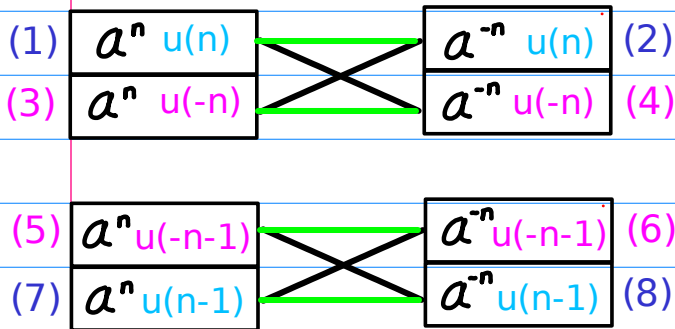
$$\begin{aligned}a^n R(n) &\longleftrightarrow a^{sh2(n)} R(-n) \\ R(n) &\longleftrightarrow \overline{R(n)} \\ a^n R(n) &\longleftrightarrow a^{sh2(n)} \overline{R(-n)}\end{aligned}$$

I.I Complementary Inverting

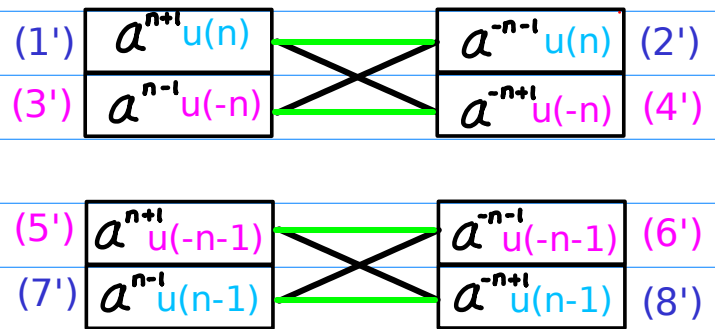
**Base Inverting
Range Complementing**

$$\begin{aligned}a^n &\longleftrightarrow a^{-n} \\ R(n) &\longleftrightarrow \overline{R(n)} \\ a^n R(n) &\longleftrightarrow a^{-n} \overline{R(n)}\end{aligned}$$

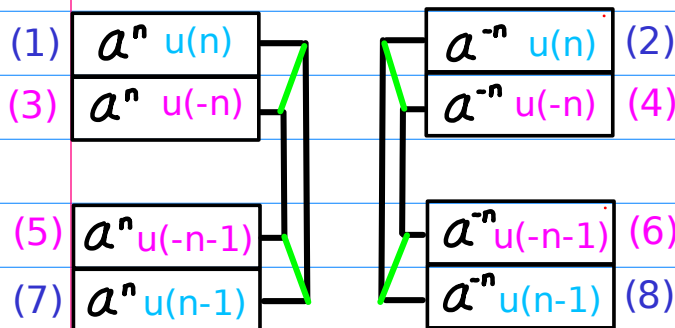
A.I Flipping Base Inverting Range Flipping



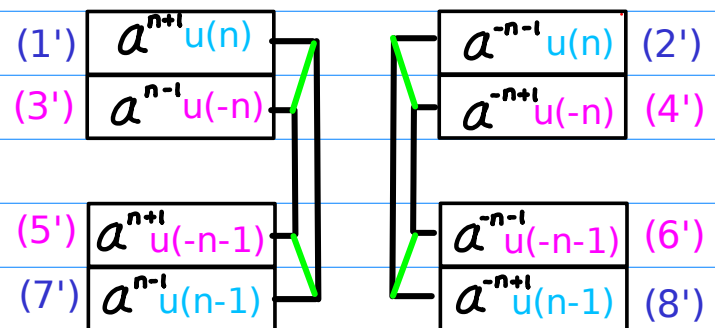
D.I Flipping2 Base Inverting Shifted Range Flipping



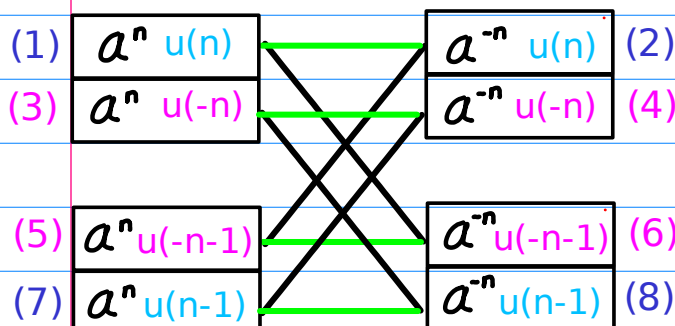
B.I Range Shifting Range Flipping Range Complementing



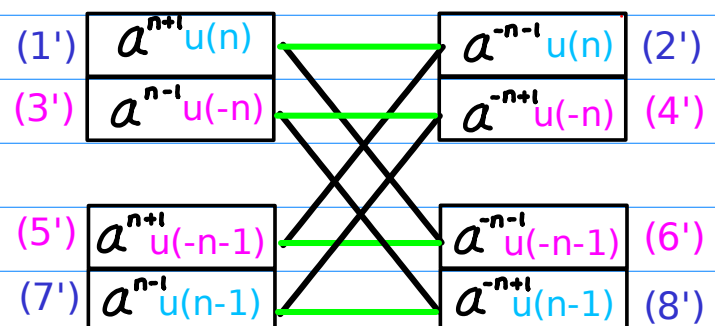
E.I Shifting2 Shifted Range Flipping Range Complementing



C.I Complementary Inverting Base Inverting Range Complementing



F.I Complementary Inverting Base Inverting Range Complementing



$$\text{shl}(b^n) \longleftrightarrow \text{shr}(b^n)$$

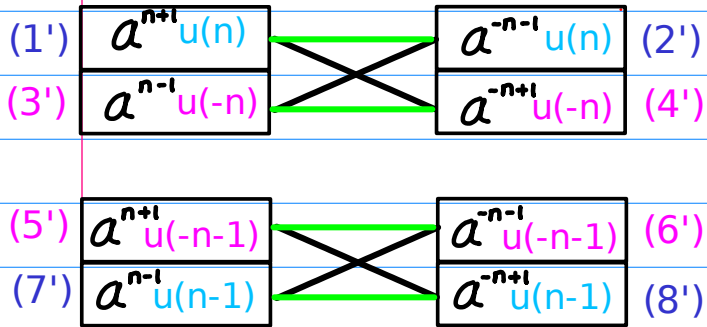
$$\text{shr}(b^n) \longleftrightarrow \text{shl}(b^n)$$

$$a^{(n+1)} \longleftrightarrow a^{(n-1)}$$

$$a^{-(n+1)} \longleftrightarrow a^{-(n-1)}$$

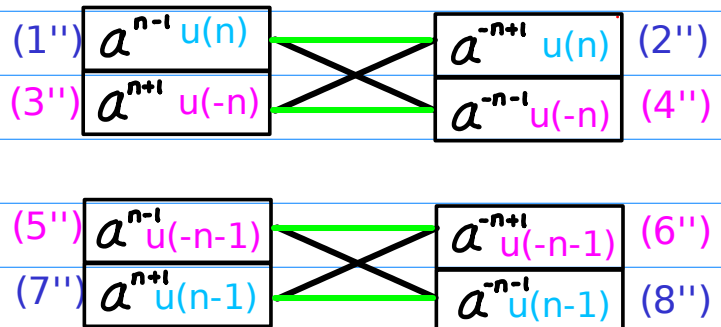
D.I Flipping2

**Base Inverting
Shifted Range Flipping**



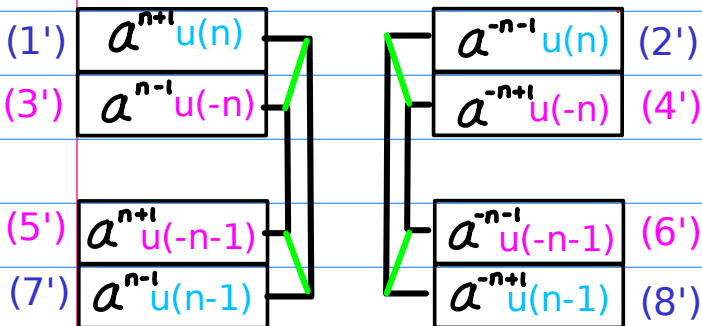
G.I Flipping2

**Base Inverting
Shifted Range Flipping**



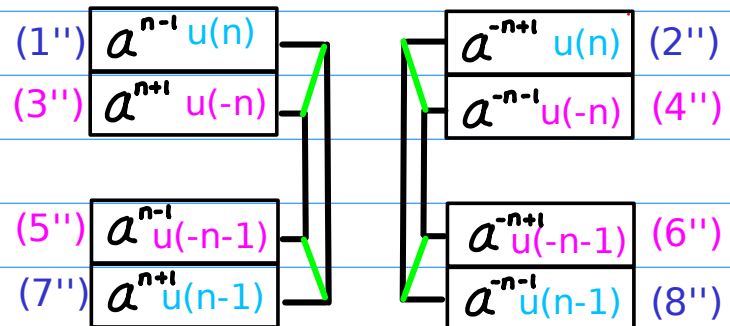
E.I Shifting2

**Shifted Range Flipping
Range Complementing**



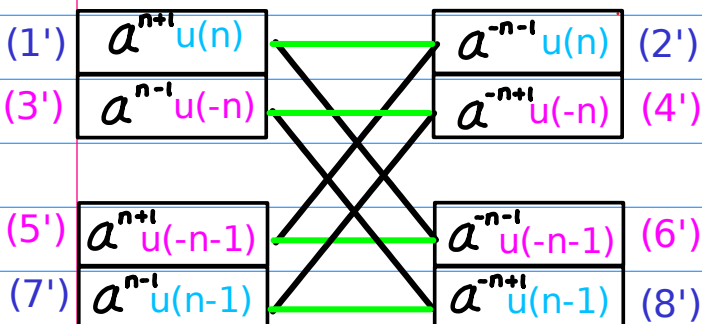
H.I Shifting2

**Shifted Range Flipping
Range Complementing**



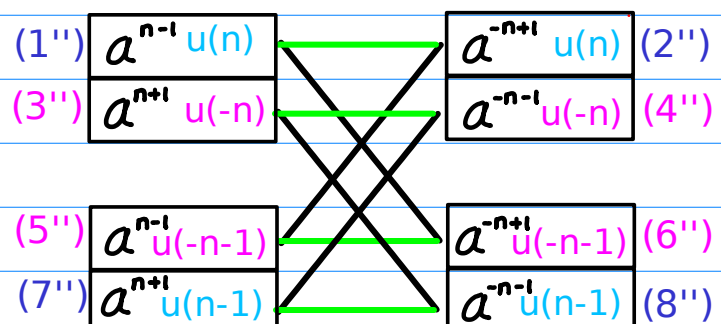
F.I Complementary Inverting

**Base Inverting
Range Complementing**



I.I Complementary Inverting

**Base Inverting
Range Complementing**



D.I Flipping2
Base Inverting
Shifted Range Flipping

G.I Flipping2
Base Inverting
Shifted Range Flipping

$$\begin{array}{cc} (1') & a^{n+l}u(n) \\ (3') & a^{n-l}u(-n) \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{cc} a^{-n-l}u(n) & (2') \\ a^{-n+l}u(-n) & (4') \end{array}$$

$$\begin{array}{cc} (1'') & a^{n-l}u(n) \\ (3'') & a^{n+l}u(-n) \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{cc} a^{-n+l}u(n) & (2'') \\ a^{-n-l}u(-n) & (4'') \end{array}$$

$$\begin{array}{cc} (5') & a^{n+l}u(-n-1) \\ (7') & a^{n-l}u(n-1) \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{cc} a^{-n-l}u(-n-1) & (6') \\ a^{-n+l}u(n-1) & (8') \end{array}$$

$$\begin{array}{cc} (5'') & a^{n-l}u(-n-1) \\ (7'') & a^{n+l}u(n-1) \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{cc} a^{-n+l}u(-n-1) & (6'') \\ a^{-n-l}u(n-1) & (8'') \end{array}$$

E.I Shifting2
Shifted Range Flipping
Range Complementing

H.I Shifting2
Shifted Range Flipping
Range Complementing

$$\begin{array}{cc} (1') & a^{n+l}u(n) \\ (5') & a^{n+l}u(-n-1) \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{cc} a^{n-l}u(-n) & (3') \\ a^{n-l}u(n-1) & (7') \end{array}$$

$$\begin{array}{cc} (1'') & a^{n-l}u(n) \\ (5'') & a^{n-l}u(-n-1) \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{cc} a^{n+l}u(-n) & (3'') \\ a^{n+l}u(n-1) & (7'') \end{array}$$

$$\begin{array}{cc} (2') & a^{-n-l}u(n) \\ (6') & a^{-n-l}u(-n-1) \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{cc} a^{-n+l}u(-n) & (4') \\ a^{-n+l}u(n-1) & (8') \end{array}$$

$$\begin{array}{cc} (2'') & a^{-n+l}u(n) \\ (6'') & a^{-n+l}u(-n-1) \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{cc} a^{-n-l}u(-n) & (4'') \\ a^{-n-l}u(n-1) & (8'') \end{array}$$

F.I Complementary Inverting
Base Inverting
Range Complementing

I.I Complementary Inverting
Base Inverting
Range Complementing

$$\begin{array}{cc} (1') & a^{n+l}u(n) \\ (5') & a^{n+l}u(-n-1) \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{cc} a^{-n-l}u(n) & (2') \\ a^{-n-l}u(-n-1) & (6') \end{array}$$

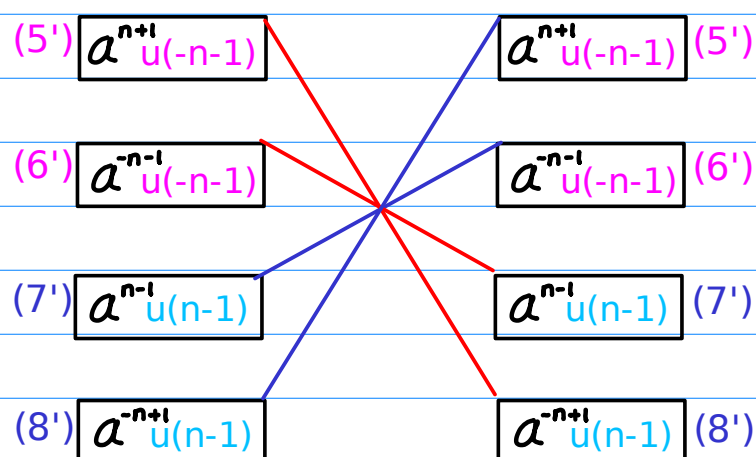
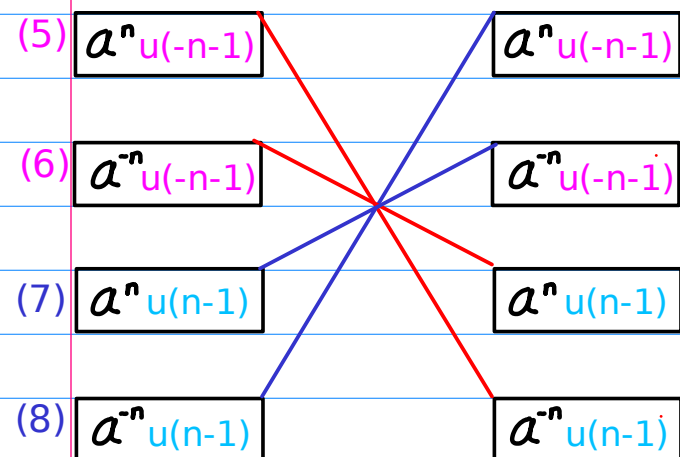
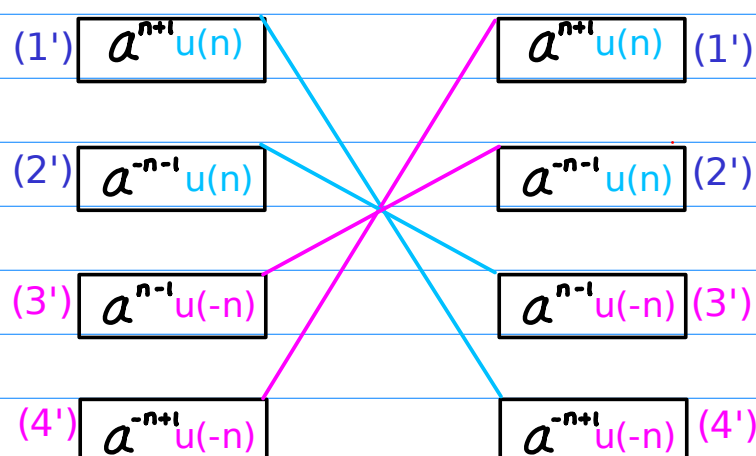
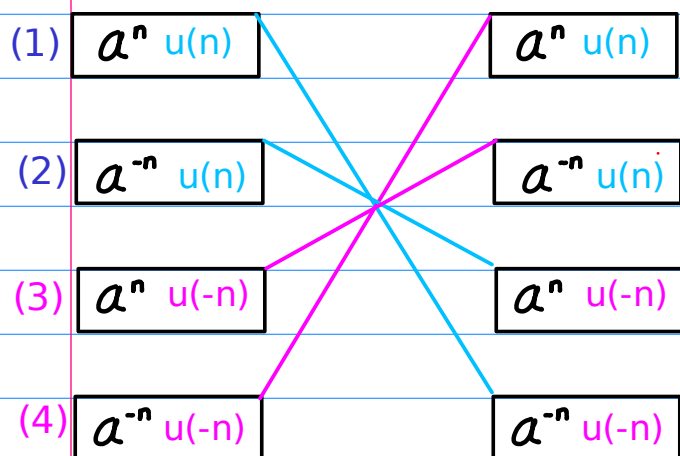
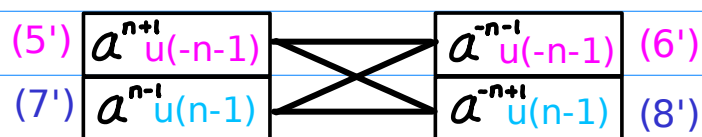
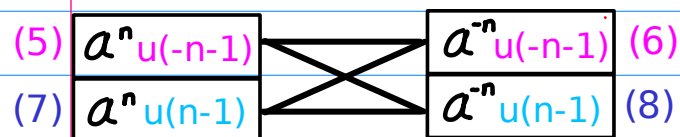
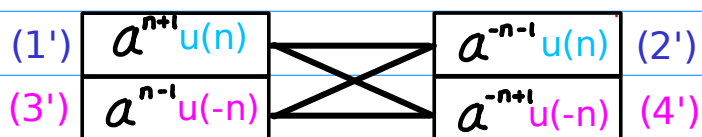
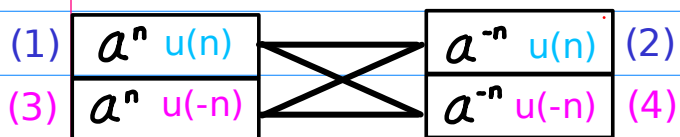
$$\begin{array}{cc} (1'') & a^{n-l}u(n) \\ (5'') & a^{n-l}u(-n-1) \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{cc} a^{-n+l}u(n) & (2'') \\ a^{-n+l}u(-n-1) & (6'') \end{array}$$

$$\begin{array}{cc} (3') & a^{n-l}u(-n) \\ (7') & a^{n-l}u(n-1) \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{cc} a^{-n+l}u(-n) & (4') \\ a^{-n+l}u(n-1) & (8') \end{array}$$

$$\begin{array}{cc} (3'') & a^{n+l}u(-n) \\ (4'') & a^{n+l}u(n-1) \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{cc} a^{-n-l}u(-n) & (4'') \\ a^{-n-l}u(n-1) & (8'') \end{array}$$

A.I Flipping Base Inverting Range Flipping

D.I Flipping2 Base Inverting Shifted Range Flipping

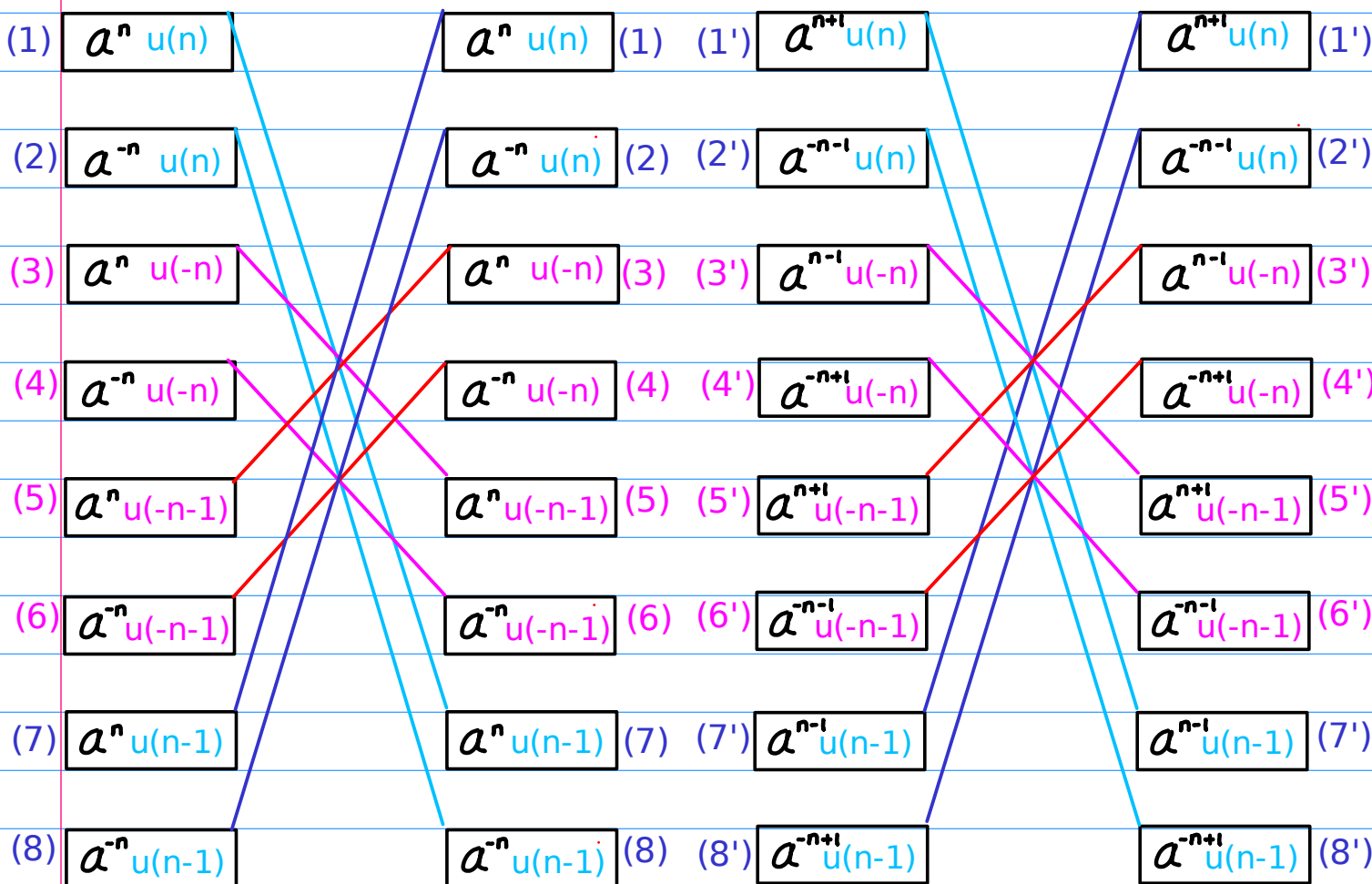
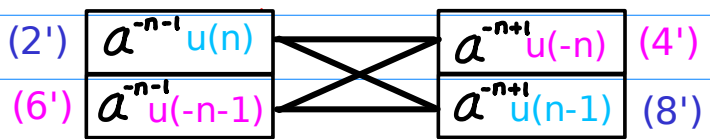
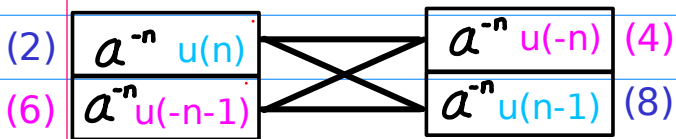
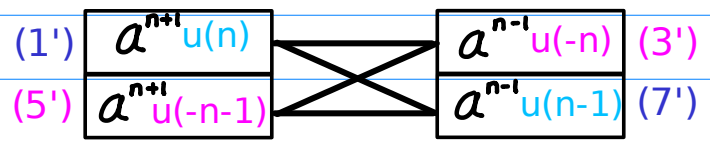
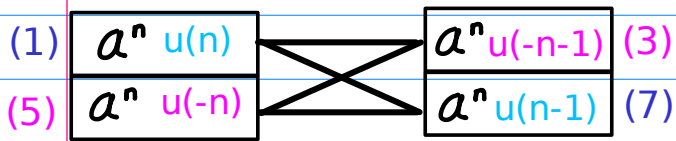


(1) - (4) (5) - (8)
 (2) - (3) (6) - (7)
 (3) - (2) (7) - (6)
 (4) - (1) (8) - (5)

(1') - (4') (5') - (8')
 (2') - (3') (6') - (7')
 (3') - (2') (7') - (6')
 (4') - (1') (8') - (5')

B.I Range Shifting Range Flipping Range Complementing

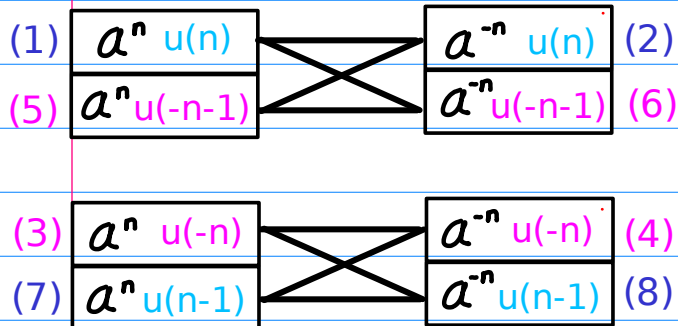
E.I Shifting2 Shifted Range Flipping Range Complementing



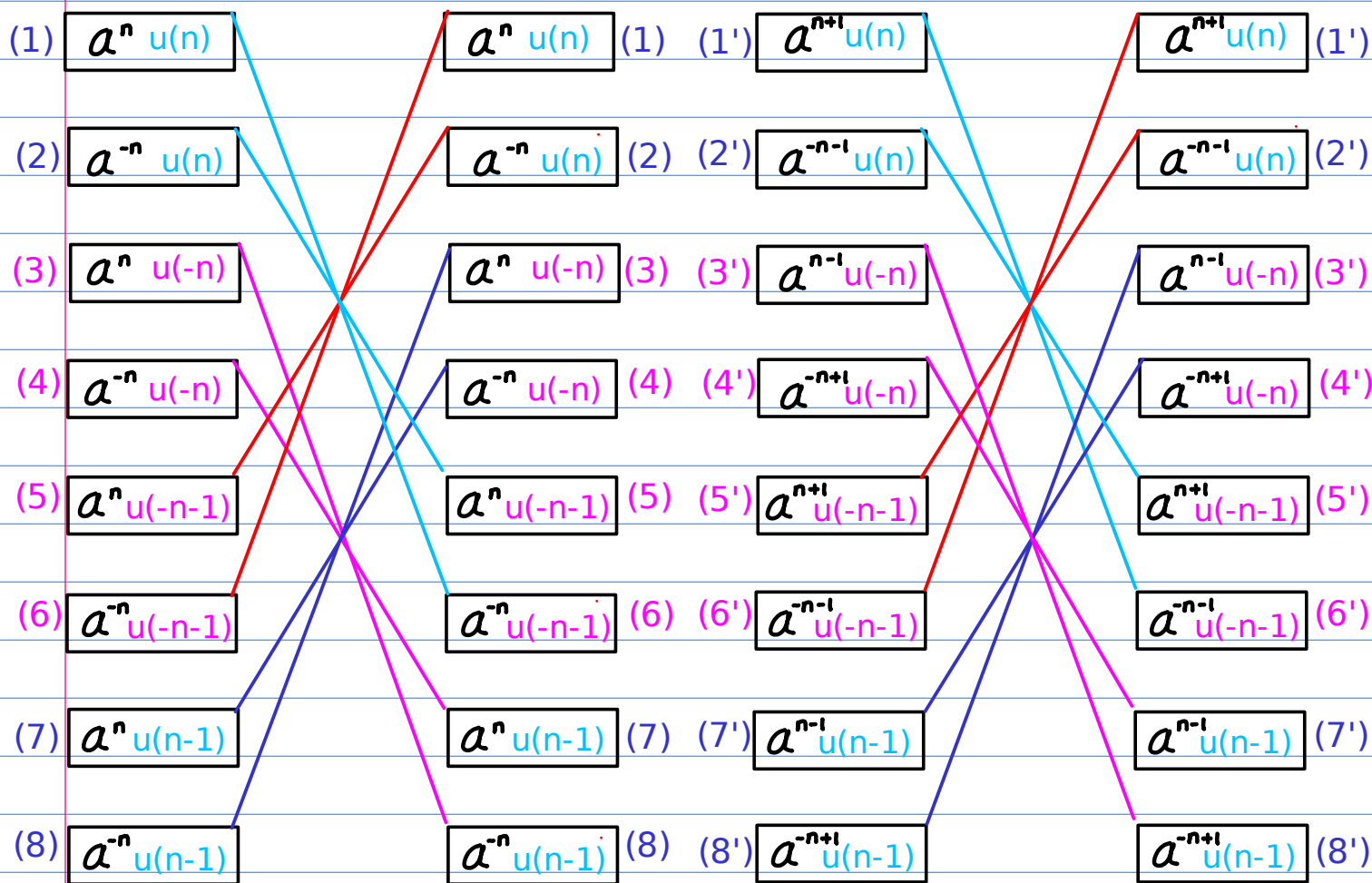
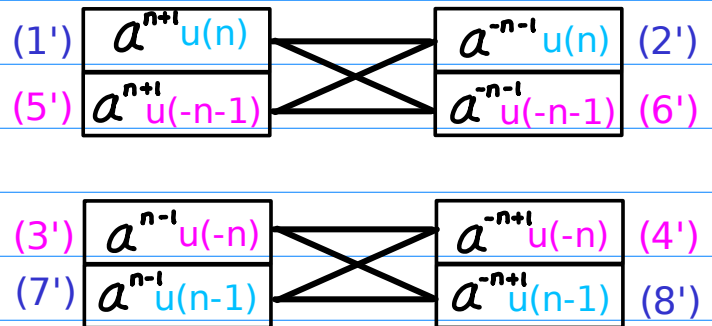
(1) - (7)
(2) - (8)
(3) - (5)
(4) - (6)
(5) - (3)
(6) - (4)
(7) - (1)
(8) - (2)

(1') - (7')
(2') - (8')
(3') - (5')
(4') - (6')
(5') - (3')
(6') - (4')
(7') - (1')
(8') - (2')

C.I Complementary Inverting Base Inverting Range Complementing

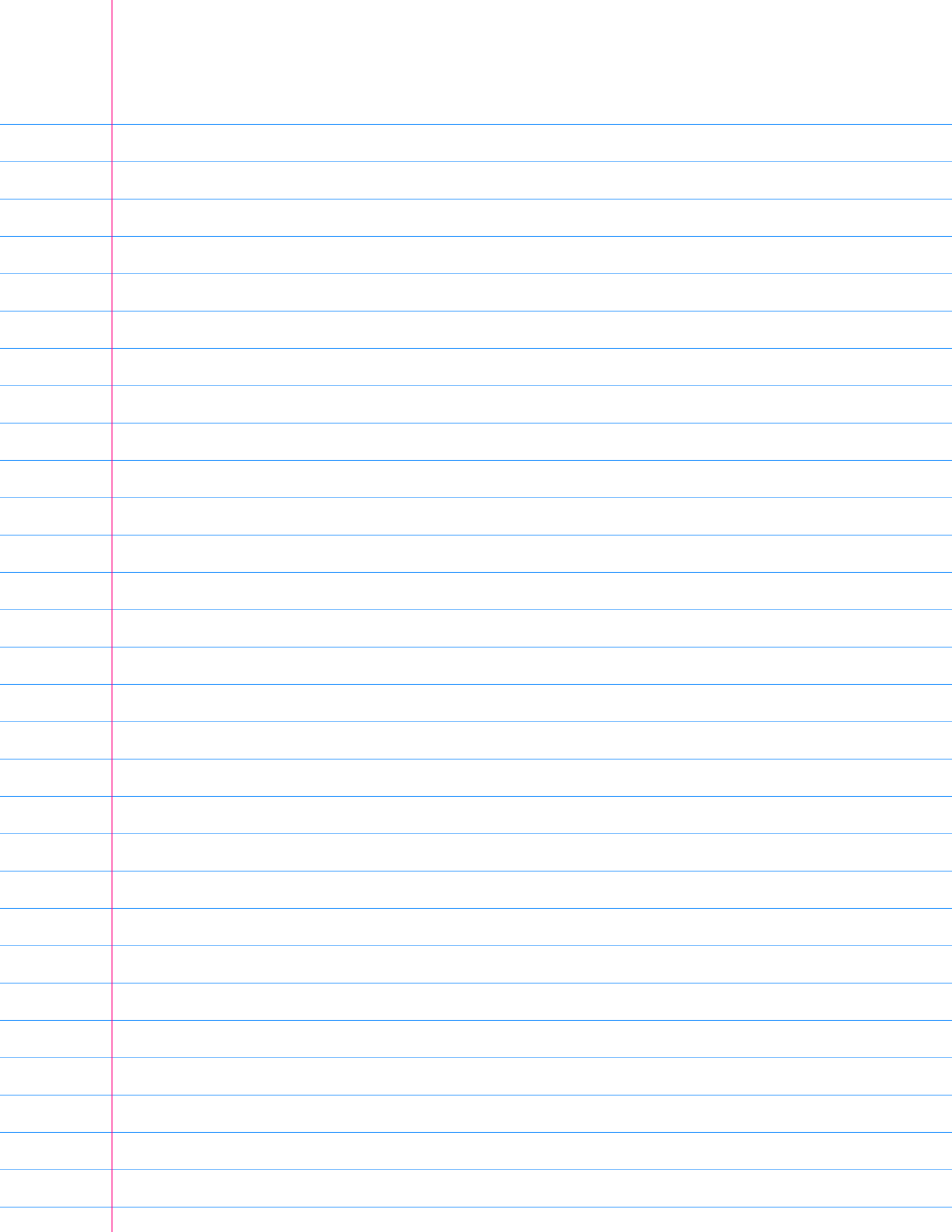


F.I Complementary Inverting Base Inverting Range Complementing



(1) - (6)
 (6) - (1)
 (2) - (5)
 (5) - (2)
 (5) - (2)
 (8) - (3)

(1') - (6')
 (6') - (1')
 (2') - (5')
 (5') - (2')
 (5') - (2')
 (8') - (3')

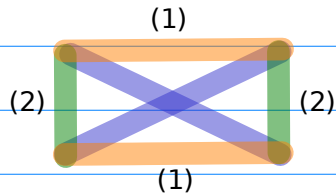


A.I Flipping

- (1) Base Inverting
- (2) Range Flipping

$$\begin{array}{cc} (1) & a^n u(n) \\ (3) & a^n u(-n) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n} u(n) & (2) \\ a^{-n} u(-n) & (4) \end{array}$$

$$\begin{array}{cc} (5) & a^n u(-n-1) \\ (7) & a^n u(n-1) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n} u(-n-1) & (6) \\ a^{-n} u(n-1) & (8) \end{array}$$



$$\begin{array}{ccc} a^n & \xleftrightarrow{(1)} & a^{-n} \\ R(n) & \xleftrightarrow{(2)} & R(-n) \\ a^n R(n) & \longleftrightarrow & a^{-n} R(-n) \end{array}$$

b^n	b^{-n}
a^n	a^{-n}
a^{-n}	a^n

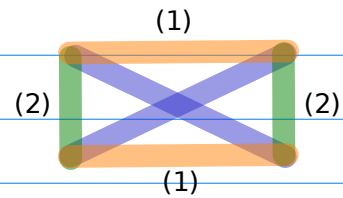
$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

D.I Flipping2

- (1) Base Inverting
- (2) Shifted Range Flipping

$$\begin{array}{cc} (1') & a^{n+1} u(n) \\ (3') & a^{n-1} u(-n) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n-1} u(n) & (2') \\ a^{-n+1} u(-n) & (4') \end{array}$$

$$\begin{array}{cc} (5') & a^{n+1} u(-n-1) \\ (7') & a^{n-1} u(n-1) \end{array} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \begin{array}{cc} a^{-n-1} u(-n-1) & (6') \\ a^{-n+1} u(n-1) & (8') \end{array}$$



$$\begin{array}{ccc} a^n & \xleftrightarrow{(1)} & a^{-n} \\ a^n R(n) & \xleftrightarrow{(2)} & a^{sh2(n)} R(-n) \\ a^n R(n) & \longleftrightarrow & a^{-sh2(n)} R(-n) \end{array}$$

b^n	$b^{-sh2(n)}$
$a^{(n+1)}$	$a^{-(n-1)}$
$a^{-(n+1)}$	$a^{(n-1)}$
$a^{(n-1)}$	$a^{-(n+1)}$
$a^{-(n-1)}$	$a^{(n+1)}$

$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

B.I Range Shifting

(1) Range Complementing

(2) Range Flipping

E.I Shifting2

(1) Shifted Range Flipping

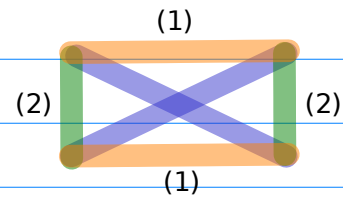
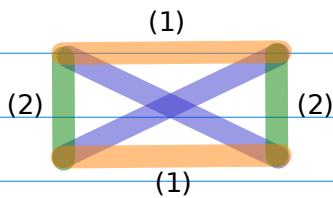
(2) Range Complementing

$$\begin{array}{l} (1) \quad a^n u(n) \\ (5) \quad a^n u(-n) \end{array} \begin{array}{c} \text{X} \\ \text{X} \end{array} \begin{array}{l} a^n u(-n-1) \quad (3) \\ a^n u(n-1) \quad (7) \end{array}$$

$$\begin{array}{l} (1') \quad a^{n+l} u(n) \\ (5') \quad a^{n+l} u(-n-1) \end{array} \begin{array}{c} \text{X} \\ \text{X} \end{array} \begin{array}{l} a^{n+l} u(-n) \quad (3') \\ a^{n+l} u(n-1) \quad (7') \end{array}$$

$$\begin{array}{l} (2) \quad a^{-n} u(n) \\ (6) \quad a^{-n} u(-n-1) \end{array} \begin{array}{c} \text{X} \\ \text{X} \end{array} \begin{array}{l} a^{-n} u(-n) \quad (4) \\ a^{-n} u(n-1) \quad (8) \end{array}$$

$$\begin{array}{l} (2') \quad a^{-n+l} u(n) \\ (6') \quad a^{-n+l} u(-n-1) \end{array} \begin{array}{c} \text{X} \\ \text{X} \end{array} \begin{array}{l} a^{-n+l} u(-n) \quad (4') \\ a^{-n+l} u(n-1) \quad (8') \end{array}$$



$$\begin{array}{l} R(n) \xleftrightarrow{(1)} R(-n) \\ R(n) \xleftrightarrow{(2)} \overline{R(n)} \\ R(n) \longleftrightarrow \overline{R(-n)} \end{array}$$

$$\begin{array}{l} a^n R(n) \xleftrightarrow{(1)} a^{sh2(n)} R(-n) \\ R(n) \xleftrightarrow{(2)} \overline{R(n)} \\ a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)} \end{array}$$

b^n	$b^{sh2(n)}$
$a^{(n+1)}$	$a^{(n-1)}$
$a^{-(n+1)}$	$a^{-(n-1)}$
$a^{(n-1)}$	$a^{(n+1)}$
$a^{-(n-1)}$	$a^{-(n+1)}$

$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

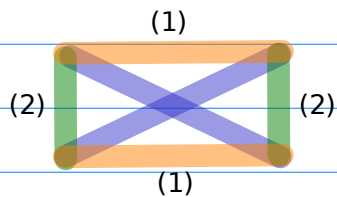
C.I Complementary Inverting

(1) Base Inverting

(2) Range Complementing

$$\begin{array}{cc} (1) & \boxed{a^n u(n)} \\ (5) & \boxed{a^n u(-n-1)} \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{cc} \boxed{a^{-n} u(n)} & (2) \\ \boxed{a^{-n} u(-n-1)} & (6) \end{array}$$

$$\begin{array}{cc} (3) & \boxed{a^n u(-n)} \\ (7) & \boxed{a^n u(n-1)} \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{cc} \boxed{a^{-n} u(-n)} & (4) \\ \boxed{a^{-n} u(n-1)} & (8) \end{array}$$



$$\begin{array}{ccc} a^n & \xleftrightarrow{(1)} & a^{-n} \\ R(n) & \xleftrightarrow{(2)} & \overline{R(n)} \\ a^n R(n) & \longleftrightarrow & a^{-n} \overline{R(n)} \end{array}$$

b^n	b^{-n}
a^n	a^{-n}
a^{-n}	a^n

$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

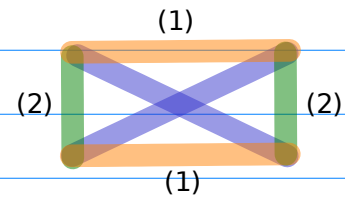
F.I Complementary Inverting

(1) Base Inverting

(2) Range Complementing

$$\begin{array}{cc} (1') & \boxed{a^{n+1} u(n)} \\ (5') & \boxed{a^{n+1} u(-n-1)} \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{cc} \boxed{a^{-n-1} u(n)} & (2') \\ \boxed{a^{-n-1} u(-n-1)} & (6') \end{array}$$

$$\begin{array}{cc} (3') & \boxed{a^{n-1} u(-n)} \\ (7') & \boxed{a^{n-1} u(n-1)} \end{array} \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \begin{array}{cc} \boxed{a^{-n+1} u(-n)} & (4') \\ \boxed{a^{-n+1} u(n-1)} & (8') \end{array}$$



$$\begin{array}{ccc} a^n & \xleftrightarrow{(1)} & a^{-n} \\ R(n) & \xleftrightarrow{(2)} & \overline{R(n)} \\ a^n R(n) & \longleftrightarrow & a^{-n} \overline{R(n)} \end{array}$$

b^n	b^{-n}
$a^{(n+1)}$	$a^{-(n+1)}$
$a^{-(n+1)}$	$a^{(n+1)}$
$a^{(n-1)}$	$a^{-(n-1)}$
$a^{-(n-1)}$	$a^{(n-1)}$

$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

$$b^n \quad b^{-sh2(n)}$$

$$a^{(n+1)} \quad a^{-(n-1)}$$

$$a^{-(n+1)} \quad a^{(n-1)}$$

$$a^{(n-1)} \quad a^{-(n+1)}$$

$$a^{-(n-1)} \quad a^{(n+1)}$$

$$b^n \quad b^{-sh2(n)}$$

$$a^{(n-1)} \quad a^{-(n+1)}$$

$$a^{-(n-1)} \quad a^{(n+1)}$$

$$a^{(n+1)} \quad a^{-(n-1)}$$

$$a^{-(n+1)} \quad a^{(n-1)}$$

$$b^n \quad b^{-(n)}$$

$$a^{(n+1)} \quad a^{-(n+1)}$$

$$a^{-(n+1)} \quad a^{(n+1)}$$

$$a^{(n-1)} \quad a^{-(n-1)}$$

$$a^{-(n-1)} \quad a^{(n-1)}$$

$$b^n \quad b^{-(n)}$$

$$a^{(n-1)} \quad a^{-(n-1)}$$

$$a^{-(n-1)} \quad a^{(n-1)}$$

$$a^{(n+1)} \quad a^{-(n+1)}$$

$$a^{-(n+1)} \quad a^{(n+1)}$$

$$b^n \quad b^{sh2(n)}$$

$$a^{(n+1)} \quad a^{(n-1)}$$

$$a^{-(n+1)} \quad a^{-(n-1)}$$

$$a^{(n-1)} \quad a^{(n+1)}$$

$$a^{-(n-1)} \quad a^{-(n+1)}$$

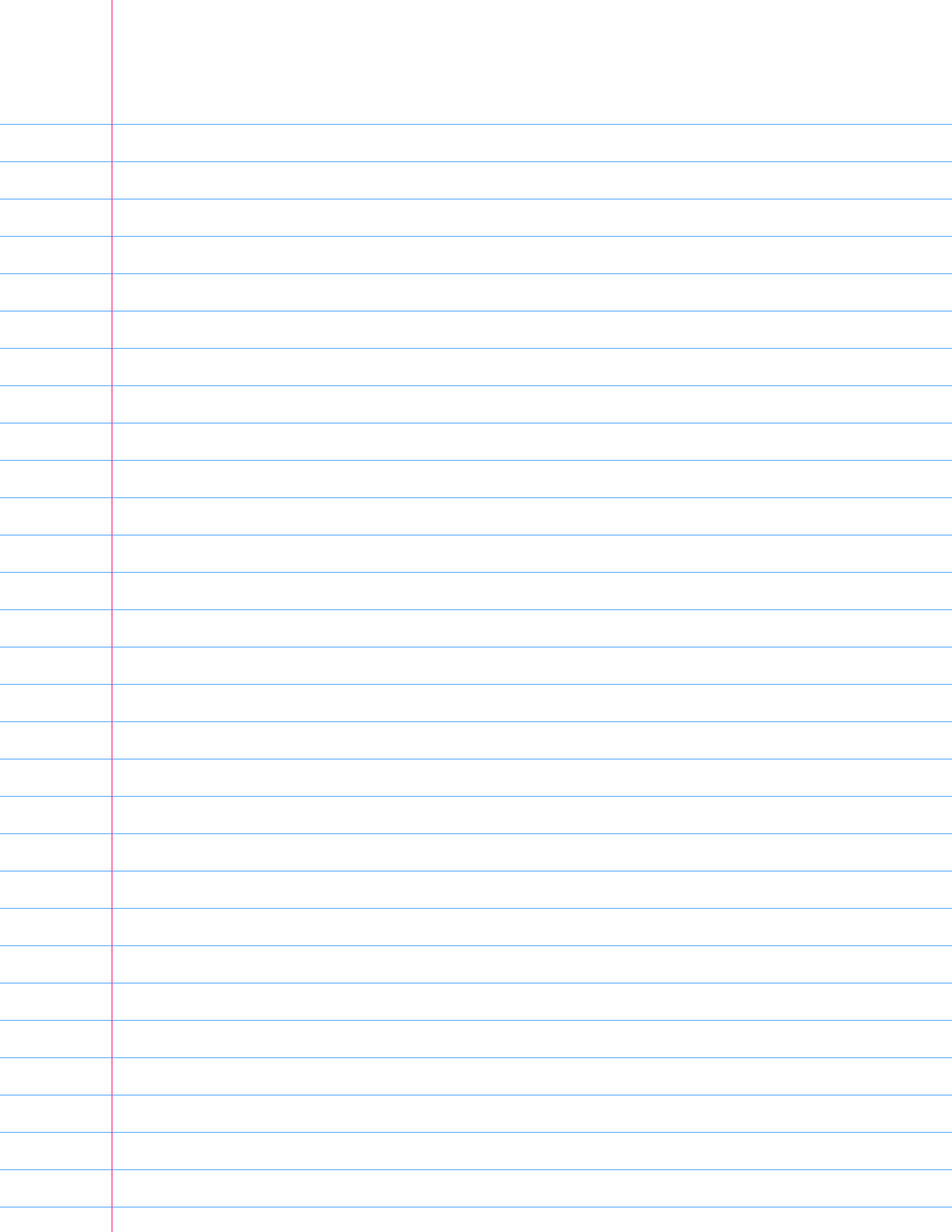
$$b^n \quad b^{sh2(n)}$$

$$a^{(n-1)} \quad a^{(n+1)}$$

$$a^{-(n-1)} \quad a^{-(n+1)}$$

$$a^{(n+1)} \quad a^{(n-1)}$$

$$a^{-(n+1)} \quad a^{-(n-1)}$$



A.I Flipping

- (1) Base Inverting
- (2) Range Flipping

$$\begin{array}{ccc}
 a^n & \xleftrightarrow{(1)} & a^{-n} \\
 R(n) & \xleftrightarrow{(2)} & R(-n) \\
 a^n R(n) & \longleftrightarrow & a^{-n} R(-n)
 \end{array}$$

D.I Flipping2

- (1) Base Inverting
- (2) Shifted Range Flipping

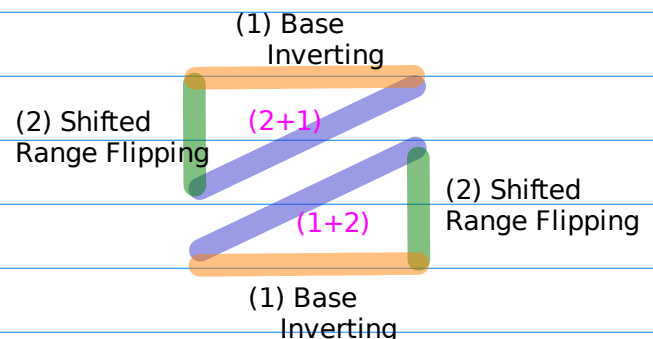
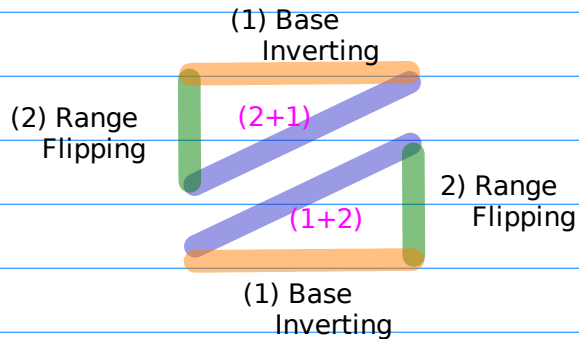
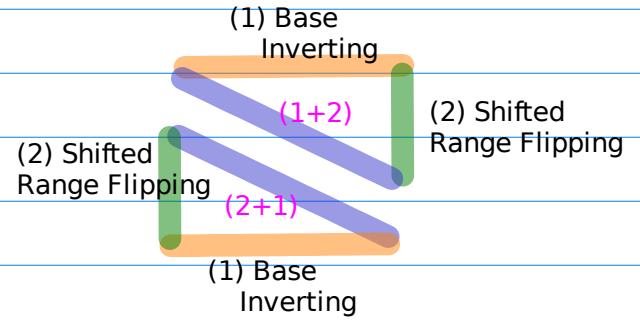
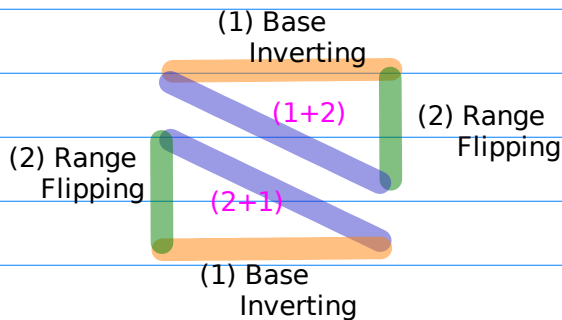
$$\begin{array}{ccc}
 a^n & \xleftrightarrow{(1)} & a^{-n} \\
 a^n R(n) & \xleftrightarrow{(2)} & a^{sh2(n)} R(-n) \\
 a^n R(n) & \longleftrightarrow & a^{-sh2(n)} R(-n)
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(1)} & a^{-n} R(n) \\
 & \xrightarrow{(2)} & a^{-n} R(-n)
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(1)} & a^{-n} R(n) \\
 & \xrightarrow{(2)} & a^{-sh2(n)} R(-n)
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(2)} & a^n R(-n) \\
 & \xrightarrow{(1)} & a^{-n} R(-n)
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(2)} & a^{sh2(n)} R(-n) \\
 & \xrightarrow{(1)} & a^{-sh2(n)} R(-n)
 \end{array}$$



B.I Range Shifting

(1) Range Complementing

(2) Range Flipping

$$R(n) \xleftrightarrow{(1)} R(-n)$$

$$R(n) \xleftrightarrow{(2)} \overline{R(n)}$$

$$R(n) \longleftrightarrow \overline{R(-n)}$$

E.I Shifting2

(1) Shifted Range Flipping

(2) Range Complementing

$$a^n R(n) \xleftrightarrow{(1)} a^{sh2(n)} R(-n)$$

$$R(n) \xleftrightarrow{(2)} \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

$$a^n R(n) \xrightarrow{(1)} a^{-n} R(n)$$

$$\xrightarrow{(2)} a^{-n} \overline{R(n)}$$

$$a^n R(n) \xrightarrow{(1)} a^{-n} R(n)$$

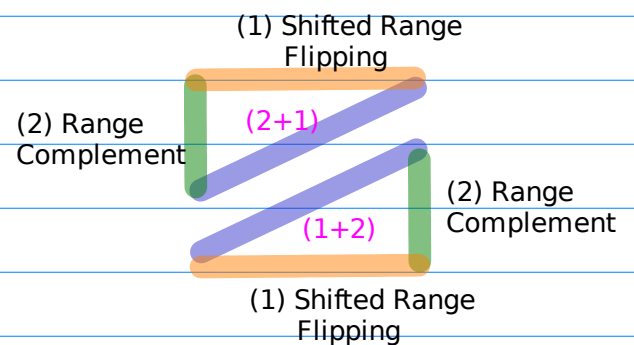
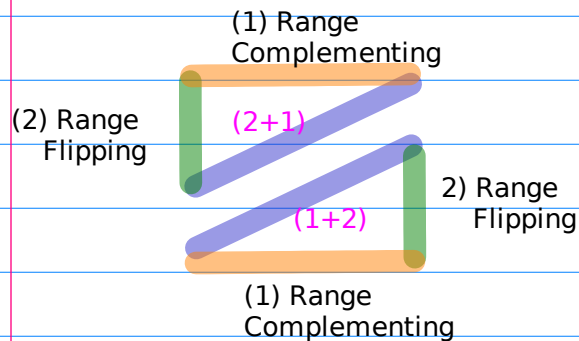
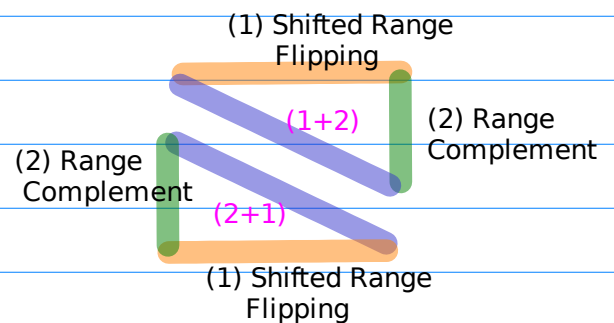
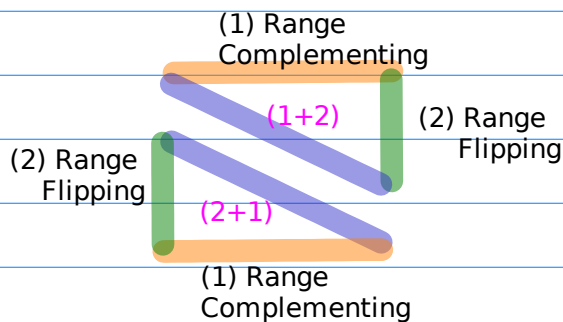
$$\xrightarrow{(2)} a^{-sh2(n)} R(-n)$$

$$a^n rng(n) \xrightarrow{(2)} a^n \overline{R(n)}$$

$$\xrightarrow{(1)} a^{-n} \overline{R(n)}$$

$$a^n R(n) \xrightarrow{(2)} a^{sh2(n)} R(-n)$$

$$\xrightarrow{(1)} a^{-sh2(n)} R(-n)$$



C.I Complementary Inverting

(1) Base Inverting

(2) Range Complementing

$$\begin{array}{ccc}
 a^n & \xleftrightarrow{(1)} & a^{-n} \\
 R(n) & \xleftrightarrow{(2)} & \overline{R(n)} \\
 a^n R(n) & \longleftrightarrow & a^{-n} \overline{R(n)}
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(1)} & a^{-n} R(n) \\
 & \xrightarrow{(2)} & a^{-n} \overline{R(n)}
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(2)} & a^n \overline{R(n)} \\
 & \xrightarrow{(1)} & a^{-n} \overline{R(n)}
 \end{array}$$

F.I Complementary Inverting

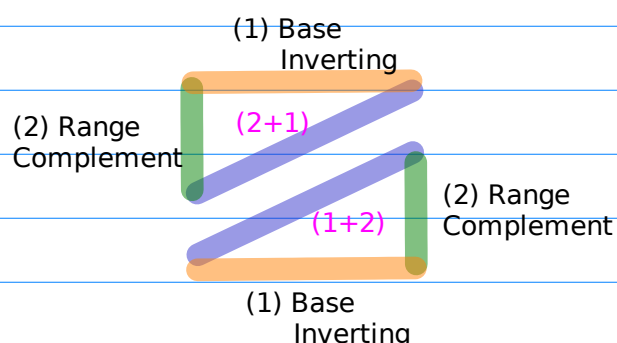
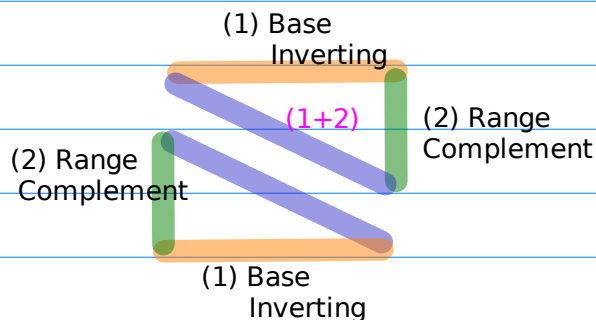
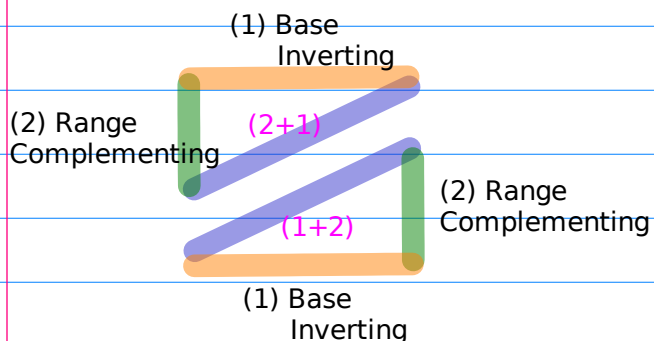
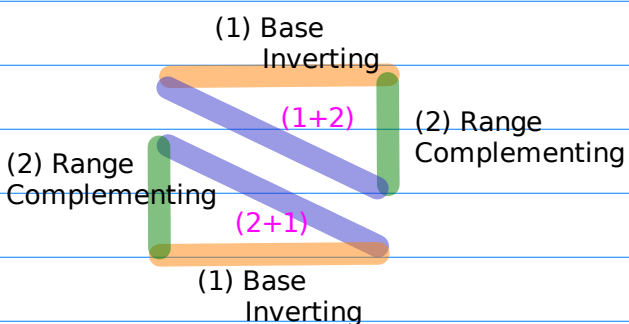
(1) Base Inverting

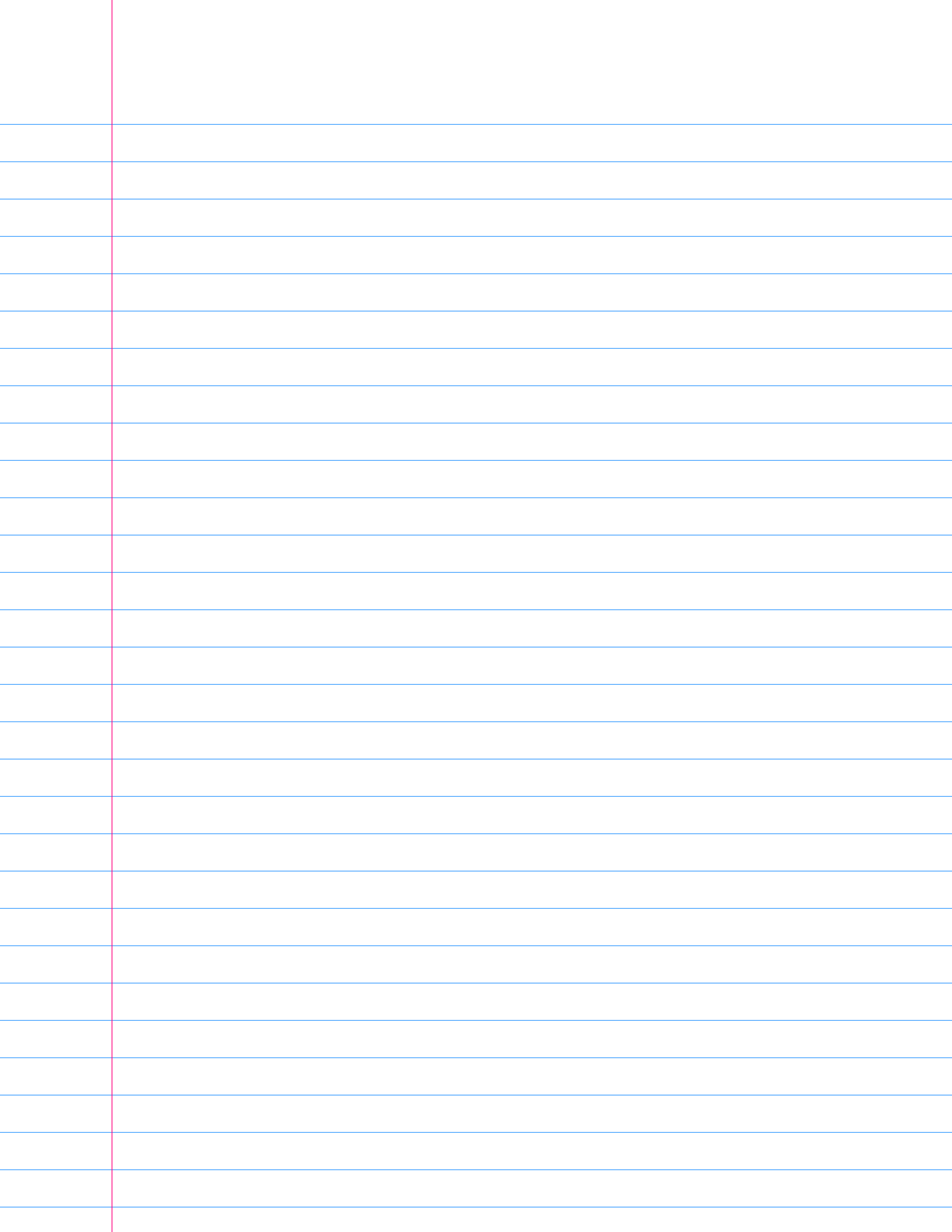
(2) Range Complementing

$$\begin{array}{ccc}
 a^n & \xleftrightarrow{(1)} & a^{-n} \\
 R(n) & \xleftrightarrow{(2)} & \overline{R(n)} \\
 a^n R(n) & \longleftrightarrow & a^{-n} \overline{R(n)}
 \end{array}$$

$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(1)} & a^{-n} R(n) \\
 & \xrightarrow{(2)} & a^{-n} \overline{R(n)}
 \end{array}$$

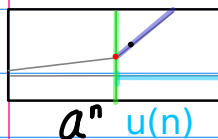
$$\begin{array}{ccc}
 a^n R(n) & \xrightarrow{(2)} & a^n \overline{R(n)} \\
 & \xrightarrow{(1)} & a^{-n} \overline{R(n)}
 \end{array}$$



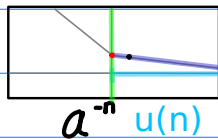


A.II Flipping Base Inverting Range Flipping

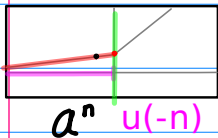
(1) 0000



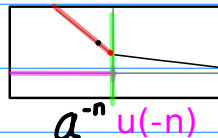
(2) 0001



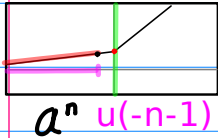
(3) 0010



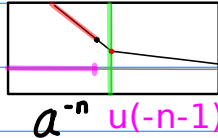
(4) 0011



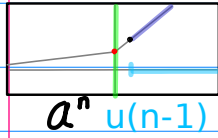
(5) 0100



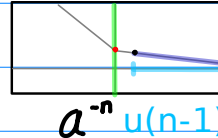
(6) 0101



(7) 0110

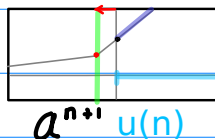


(8) 0111

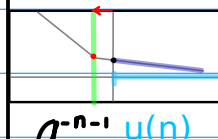


D.II Flipping2 Base Inverting Shifted Range Flipping

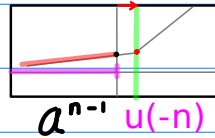
(1') 1000



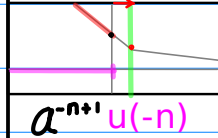
(2') 1001



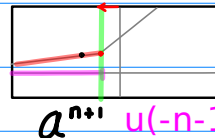
(3') 1010



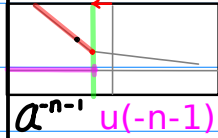
(4') 1011



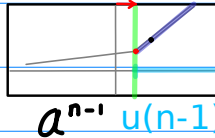
(5') 1100



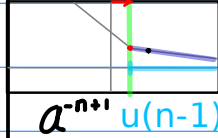
(6') 1101



(7') 1110



(8') 1111



Shifted Range Flipping
= Exponent Shifting2
+ Range Flipping

$$a^n R(n) \longleftrightarrow a^{-n} R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-sh2(n)} R(-n)$$

$$a^n \longleftrightarrow a^{-n}$$

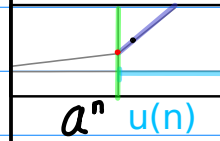
$$a^n \longleftrightarrow a^{-n}$$

$$R(n) \longleftrightarrow R(-n)$$

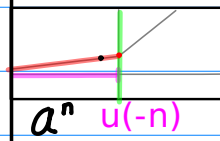
$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

B.II Range Shifting Range Flipping Range Complementing

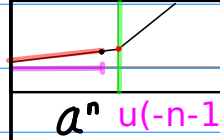
(1) 0000



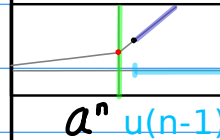
(3) 0010



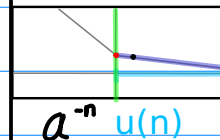
(5) 0100



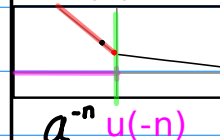
(7) 0110



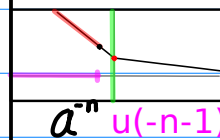
(2) 0001



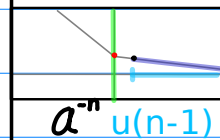
(4) 0011



(6) 0101



(8) 0111



Range Shifting
= Range Flipping
+ Range Complementing

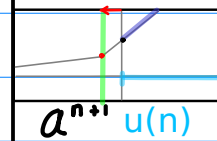
$$R(n) \longleftrightarrow R(-n)$$

$$R(n) \longleftrightarrow R(-n)$$

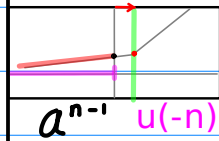
$$R(n) \longleftrightarrow \overline{R(n)}$$

E.II Shifting2 Shifted Range Flipping Range Complementing

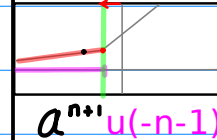
(1') 1000



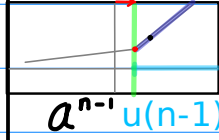
(3') 1010



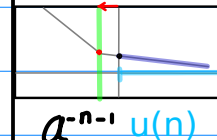
(5') 1100



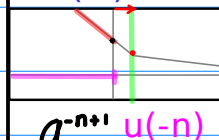
(7') 1110



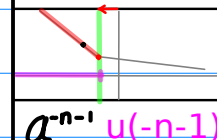
(2') 1001



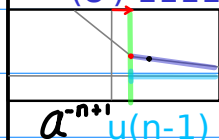
(4') 1011



(6') 1101



(8') 1111



Shifted Range Flipping
= Exponent Shifting2
+ Range Flipping

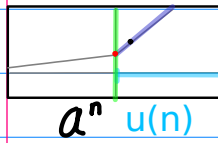
$$a^n R(n) \longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

$$a^n R(n) \longleftrightarrow a^{sh2(n)} R(-n)$$

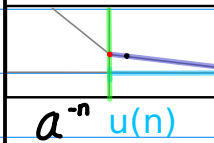
$$R(n) \longleftrightarrow \overline{R(n)}$$

C.II Complementary Inverting Base Inverting Range Complementing

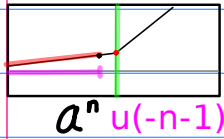
(1) 0000



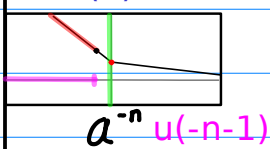
(2) 0001



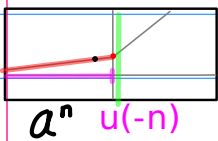
(5) 0100



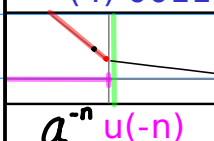
(6) 0101



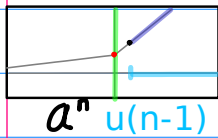
(3) 0010



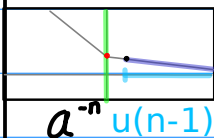
(4) 0011



(7) 0110



(8) 0111

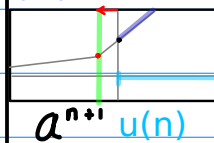


$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

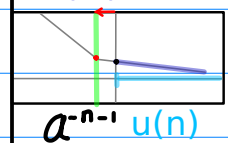
$$\begin{aligned} a^n &\longleftrightarrow a^{-n} \\ R(n) &\longleftrightarrow \overline{R(n)} \end{aligned}$$

F.II Complementary Inverting Base Inverting Range Complementing

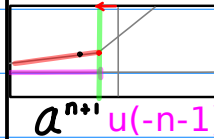
(1') 1000



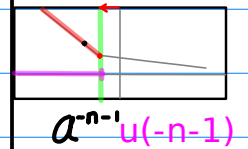
(2') 1001



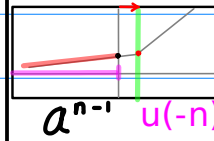
(5') 1100



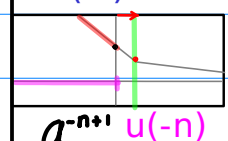
(6') 1101



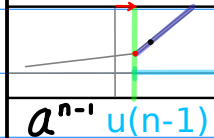
(3') 1010



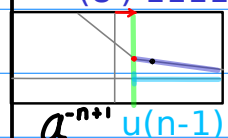
(4') 1011



(7') 1110

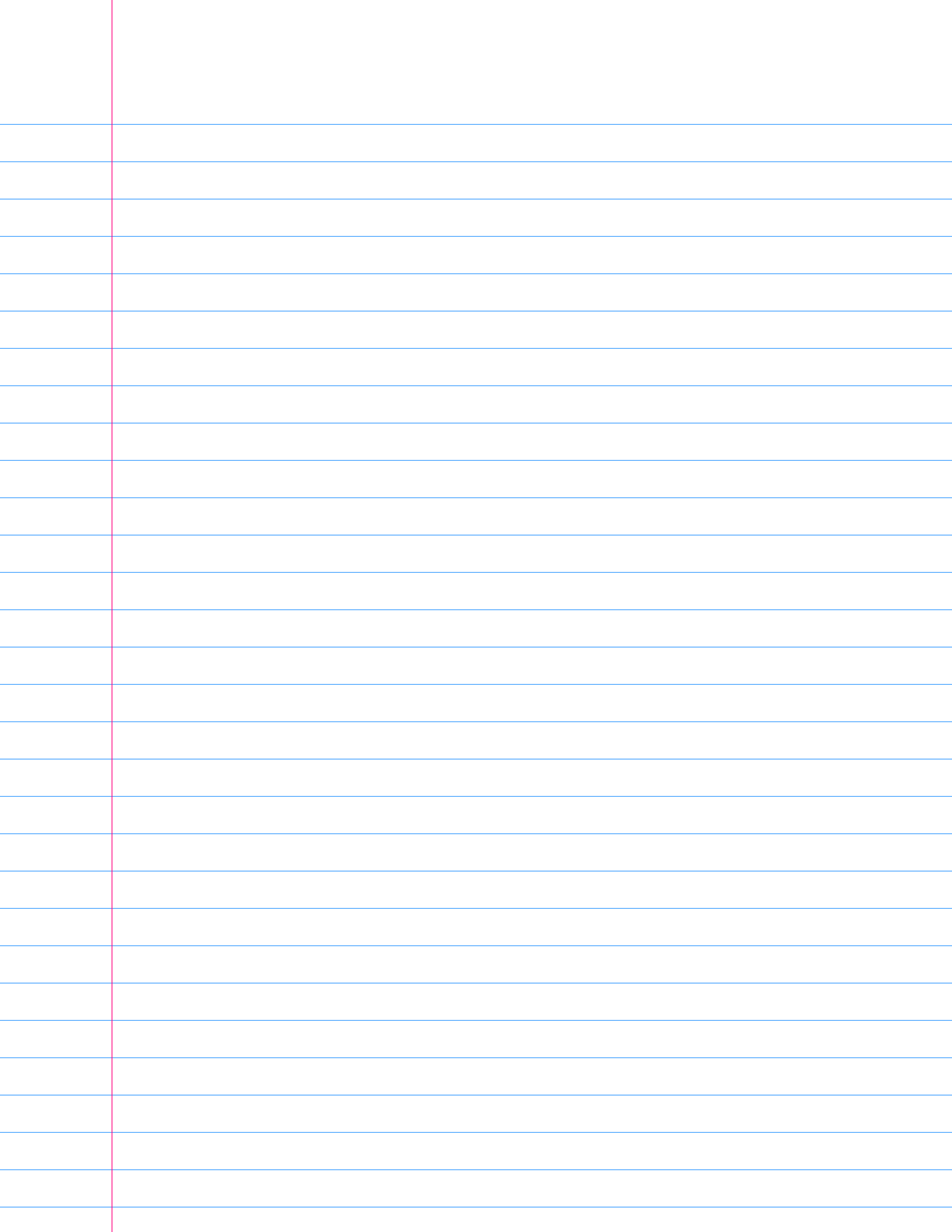


(8') 1111



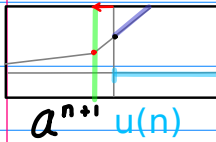
$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

$$\begin{aligned} a^n &\longleftrightarrow a^{-n} \\ R(n) &\longleftrightarrow \overline{R(n)} \end{aligned}$$

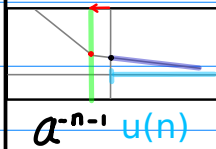


D.II Flipping2 Base Inverting Shifted Range Flipping

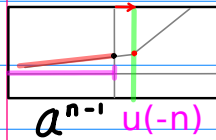
(1') 1000



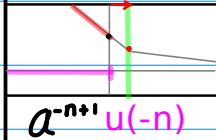
(2') 1001



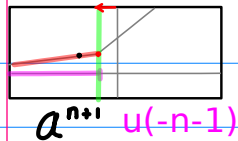
(3') 1010



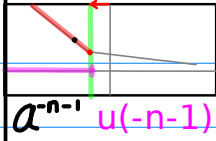
(4') 1011



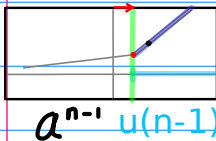
(5') 1100



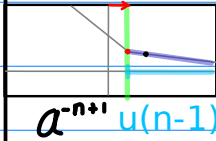
(6') 1101



(7') 1110



(8') 1111



Shifted Range Flipping
= Exponent Shifting2
+ Range Flipping

$$n+1 \xleftrightarrow{\text{sh } 2(n)} n-1$$

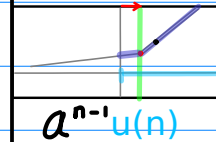
$$a^n \longleftrightarrow a^{-n}$$

$$a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} R(-n)$$

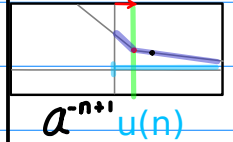
$$a^n R(n) \longleftrightarrow a^{-\text{sh } 2(n)} R(-n)$$

G.II Flipping2 Base Inverting Shifted Range Flipping

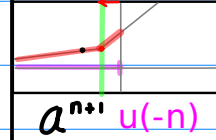
(1'') 1000



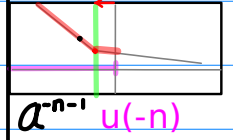
(2'') 1001



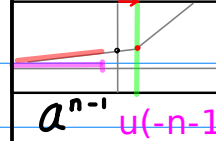
(3'') 1010



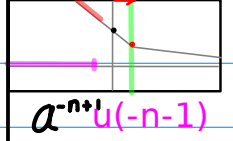
(4'') 1011



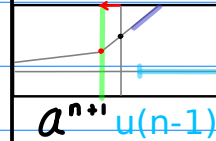
(5'') 1100



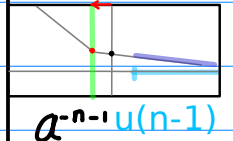
(6'') 1101



(7'') 1110



(8'') 1111



Shifted Range Flipping
= Exponent Shifting2
+ Range Flipping

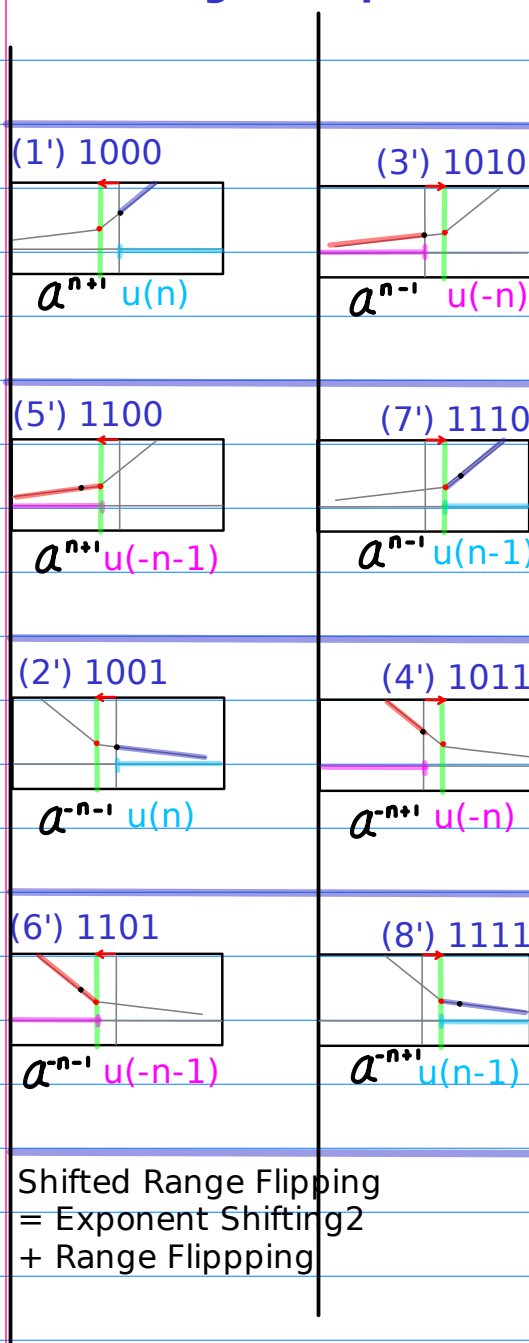
$$n+1 \xleftrightarrow{\text{sh } 2(n)} n-1$$

$$a^n \longleftrightarrow a^{-n}$$

$$a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} R(-n)$$

$$a^n R(n) \longleftrightarrow a^{-\text{sh } 2(n)} R(-n)$$

E.II Shifting2 Shifted Range Flipping Range Complementing



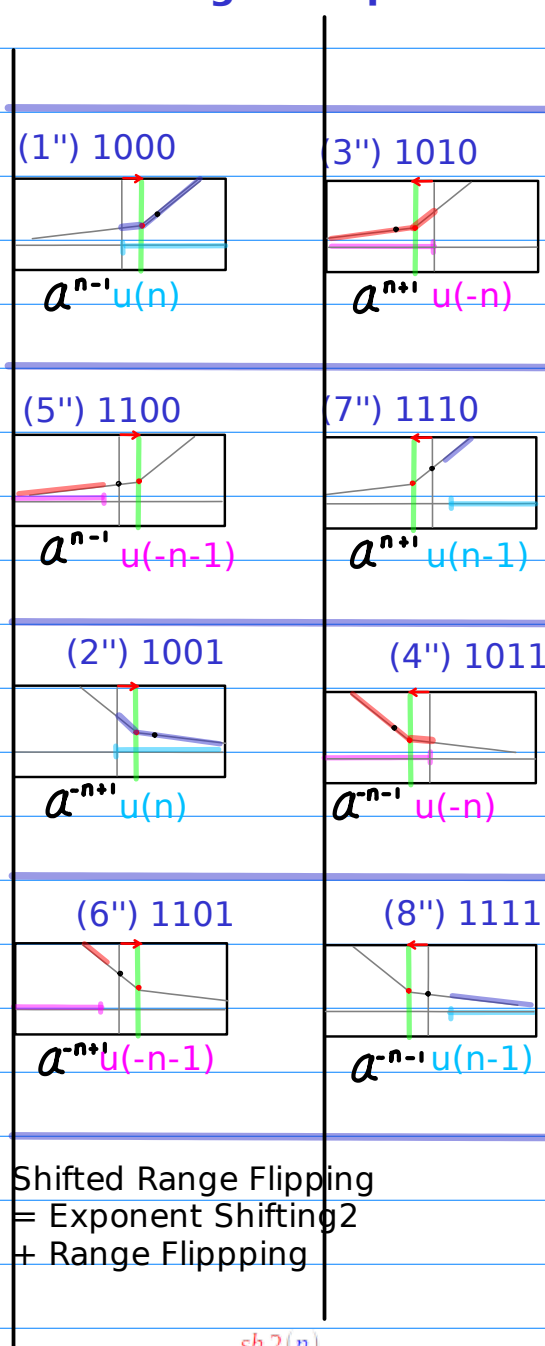
$$n+1 \xleftrightarrow{\text{sh } 2(n)} n-1$$

$$a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} R(-n) \quad \boxed{\times} \quad a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} R(-n)$$

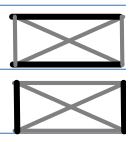
$$R(n) \longleftrightarrow \overline{R(n)} \quad \boxed{\times} \quad R(n) \longleftrightarrow \overline{R(n)}$$

$$a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} \overline{R(-n)} \quad \boxed{\times} \quad a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} \overline{R(-n)}$$

H.II Shifting2 Shifted Range Flipping Range Complementing



$$n+1 \xleftrightarrow{\text{sh } 2(n)} n-1$$



$$a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} R(-n)$$

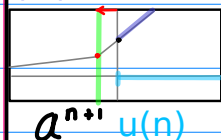
$$R(n) \longleftrightarrow \overline{R(n)}$$



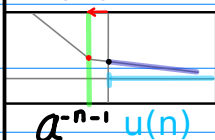
$$a^n R(n) \longleftrightarrow a^{\text{sh } 2(n)} \overline{R(-n)}$$

F.II Complementary Inverting Base Inverting Range Complementing

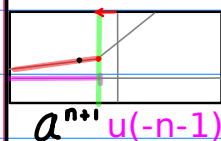
(1') 1000



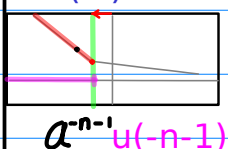
(2') 1001



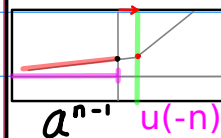
(5') 1100



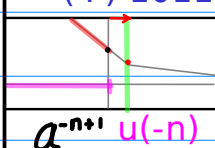
(6') 1101



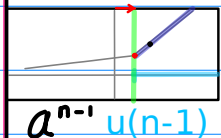
(3') 1010



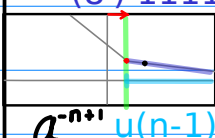
(4') 1011



(7') 1110

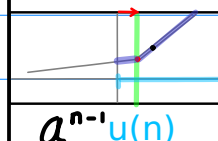


(8') 1111

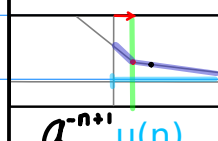


I.II Complementary Inverting Base Inverting Range Complementing

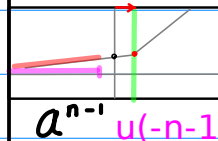
(1'') 1000



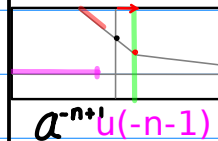
(2'') 1001



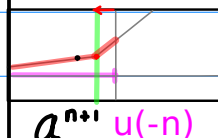
(5'') 1100



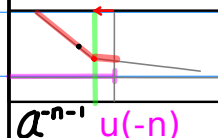
(6'') 1101



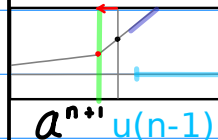
(3'') 1010



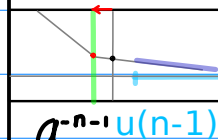
(4'') 1011



(7'') 1110

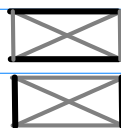


(8'') 1111



Shifted Range Flipping
= Exponent Shifting²
+ Range Flipping

$$\begin{array}{ccc} a^n & \longleftrightarrow & a^{-n} \\ R(n) & \longleftrightarrow & \overline{R(n)} \end{array}$$



$$\begin{array}{ccc} a^n & \longleftrightarrow & a^{-n} \\ R(n) & \longleftrightarrow & \overline{R(n)} \end{array}$$

$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$



$$a^n R(n) \longleftrightarrow a^{-n} \overline{R(n)}$$

Shifted Sequence 1 x'

