Laurent Series and z-Transform - Geometric Series Permutations B

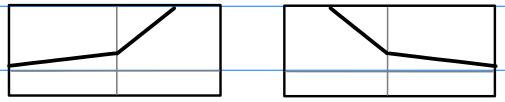
20251103 Mon

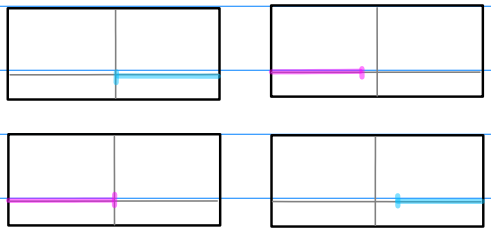
a^n × $R(n)$

a^n	a^{-n}

×

u(n)	u(-n-1)
u(-n)	u(n-1)



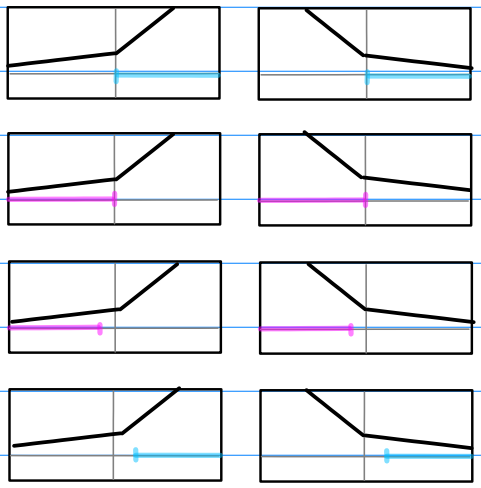


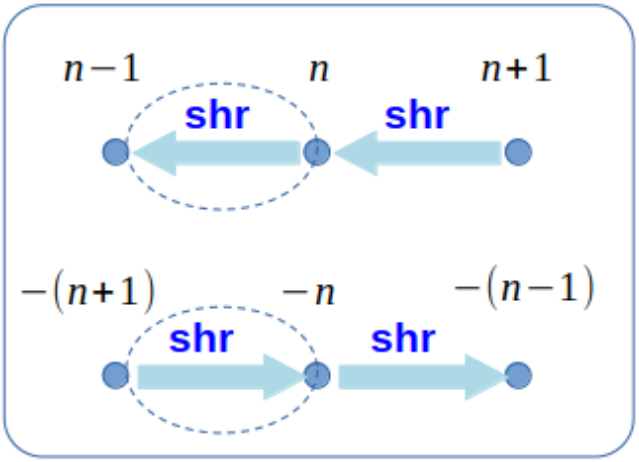
(1) a^n u(n) a^{-n} u(n) (2)

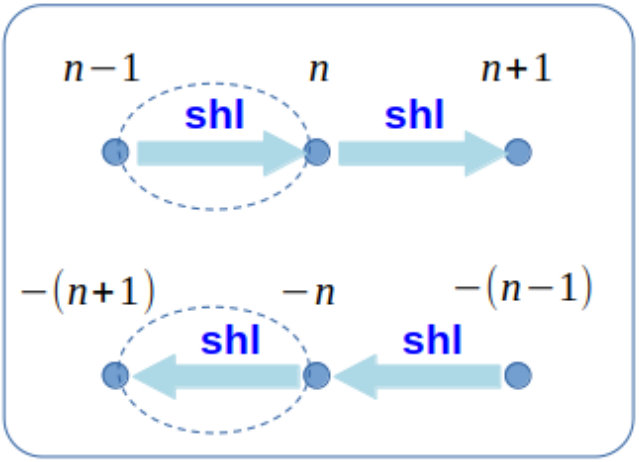
(3) a^n u(-n) a^{-n} u(-n) (4)

(5) a^n u(-n-1) a^{-n} u(-n-1) (6)

(7) a^n u(n-1) a^{-n} u(n-1) (8)







shr $n \leftarrow n-1$

replace **n** with **n − 1**

srl $n \leftarrow n+1$

replace **n** with **n + 1**

$f(n-1)$ **shr** $f(n)$

$f(n)$ **shr** $f(n+1)$

$f(n)$ **shl** $f(n-1)$

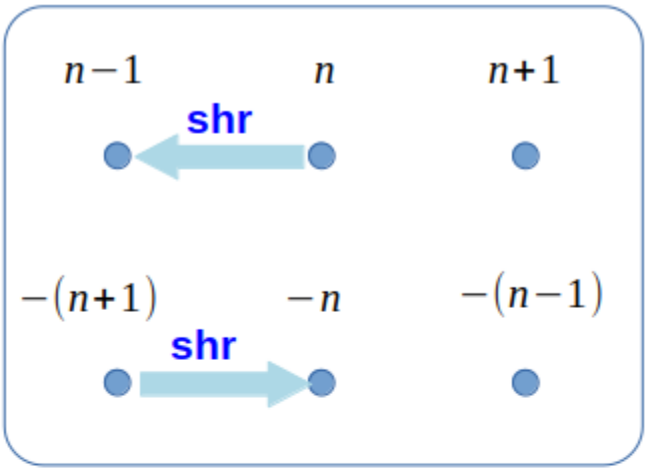
$f(n+1)$ **shl** $f(n)$

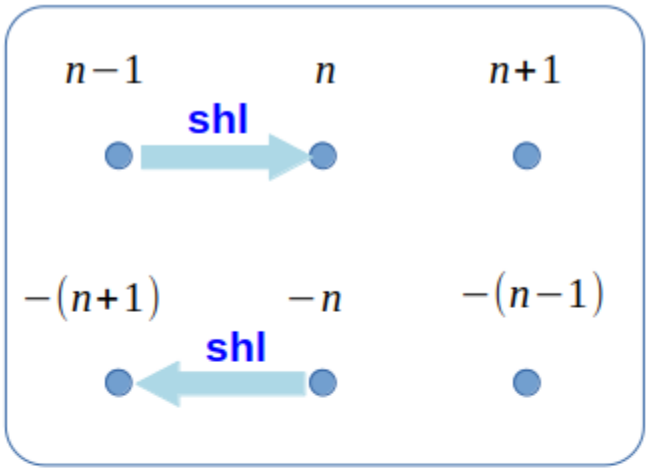
$f(-n)$ **shr** $f(-n-1)$

$f(-n+1)$ **shr** $f(-n)$

$f(-n-1)$ **shl** $f(-n)$

$f(-n)$ **shl** $f(-n+1)$





shr $n \leftarrow n-1$

replace **n** with **n − 1**

srl $n \leftarrow n+1$

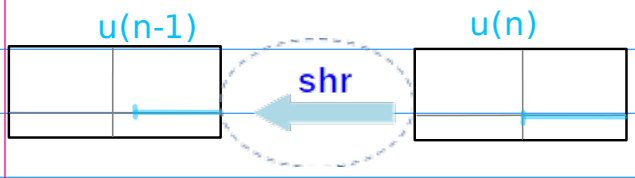
replace **n** with **n + 1**

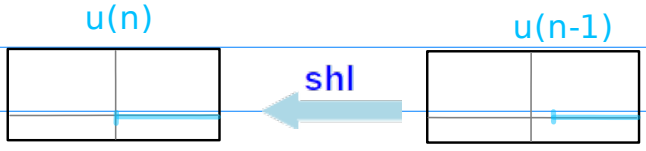
$f(n-1)$ **shr** $f(n)$

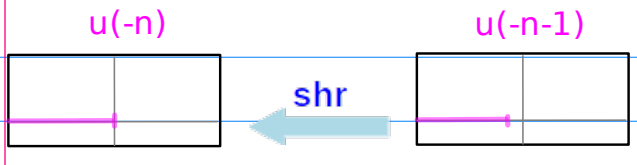
$f(-n)$ **shr** $f(-n-1)$

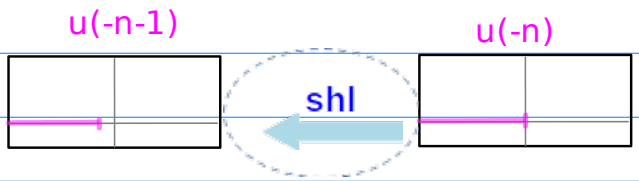
$f(n)$ **shl** $f(n-1)$

$f(-n-1)$ **shl** $f(-n)$

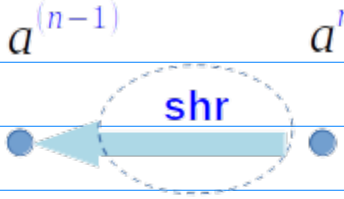


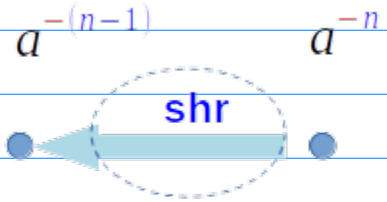




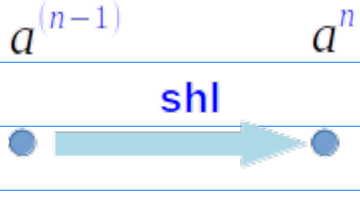


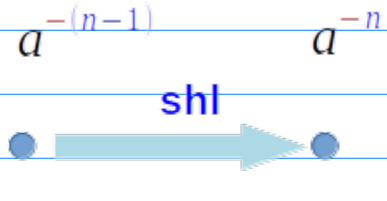
shr $n \leftarrow n-1$

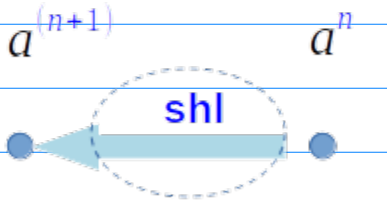


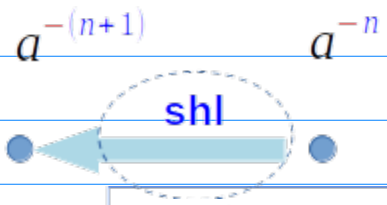


srl $n \leftarrow n+1$

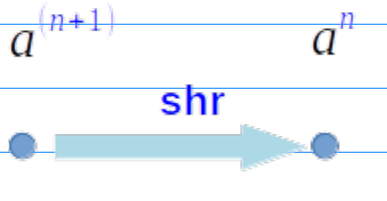


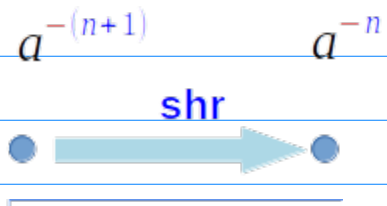






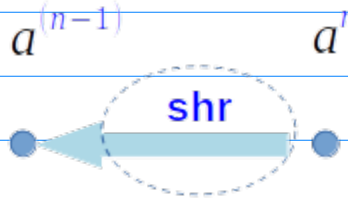
srl $n \leftarrow n+1$

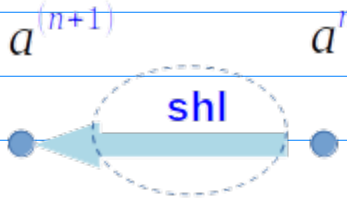


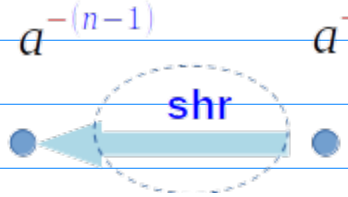


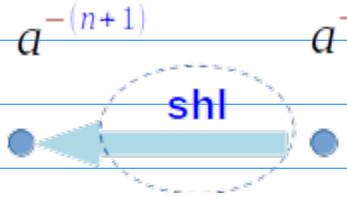
shr $n \leftarrow n-1$

Exponent Shifting







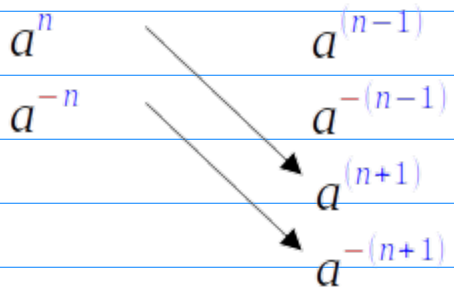


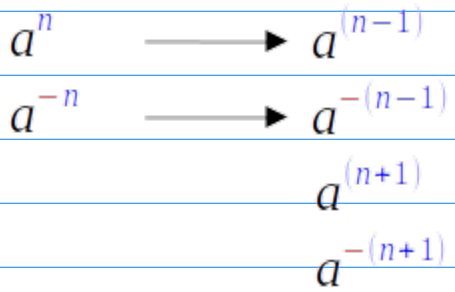
$$shl(b^n) = b^{(n+1)}$$

b^n	b^{n+1}
a^n	a^{n+1}
a^{-n}	$a^{-(n+1)}$

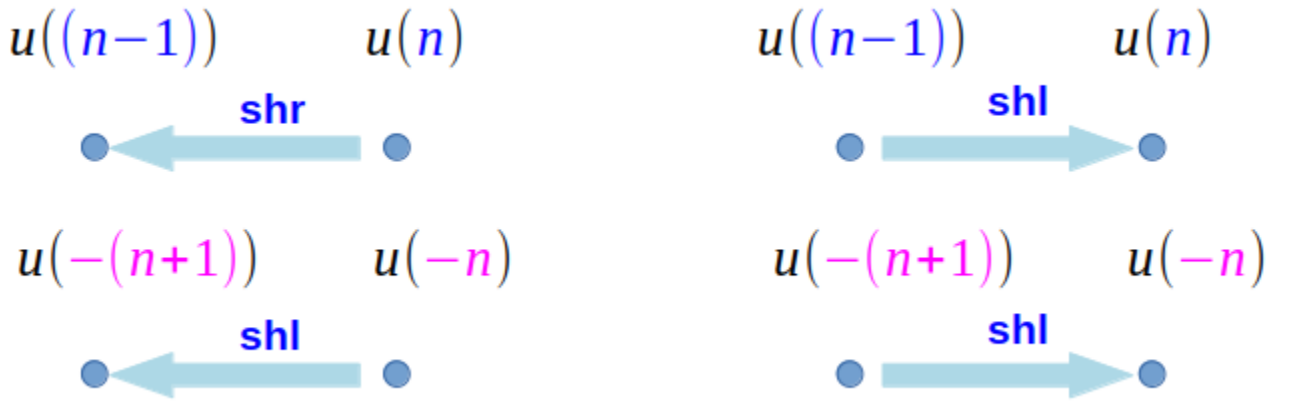
$$shr(b^n) = b^{(n-1)}$$

b^n	b^{n-1}
a^n	a^{n-1}
a^{-n}	$a^{-(n-1)}$





Range Shifting

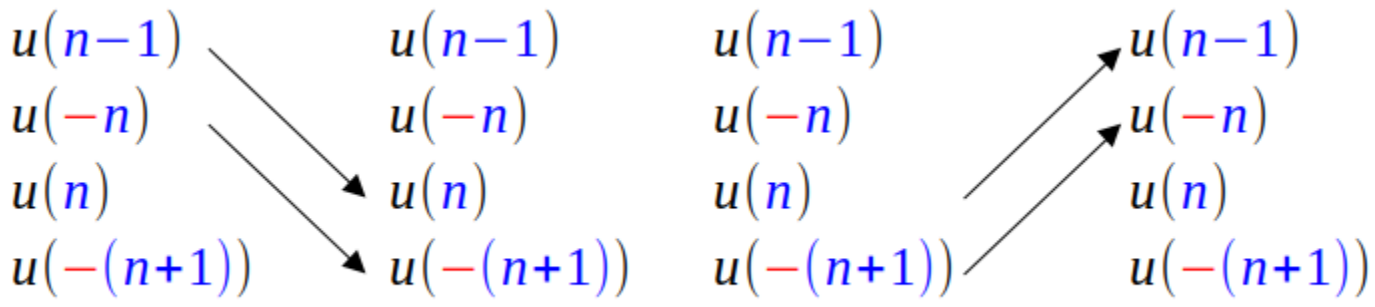


$shl(R(n)) = R(n+1)$

$R(n)$	$R(n+1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-(n+1))$

$shr(R(n)) = R(n-1)$

$R(n)$	$R(n-1)$
$u(n)$	$u(n-1)$
$u(-(n+1))$	$u(-n)$



$shl(b^n)$

$b^n \rightarrow b^{n+1}$

$shl(b^n) = b^{(n+1)}$

b^n	b^{n+1}
a^n	a^{n+1}
a^{-n}	$a^{-(n+1)}$

$shr(b^n)$

$b^n \rightarrow b^{n-1}$

$shr(b^n) = b^{(n-1)}$

b^n	b^{n-1}
a^n	a^{n-1}
a^{-n}	$a^{-(n-1)}$

$shl(R(n))$

$R(n) \rightarrow R(n+1)$

$shl(R(n)) = R(n+1)$

$R(n)$	$R(n+1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-(n+1))$

$shr(R(n))$

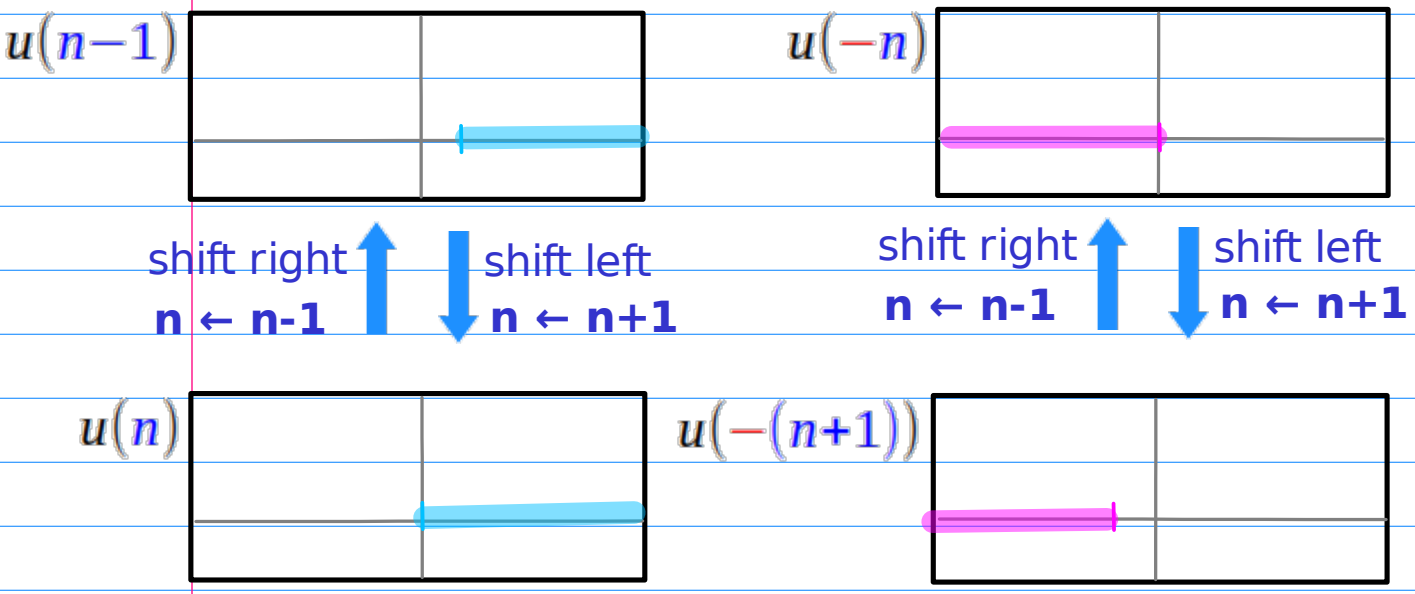
$R(n) \rightarrow R(n-1)$

$shr(R(n)) = R(n-1)$

$R(n)$	$R(n-1)$
$u(n)$	$u(n-1)$
$u(-(n+1))$	$u(-n)$

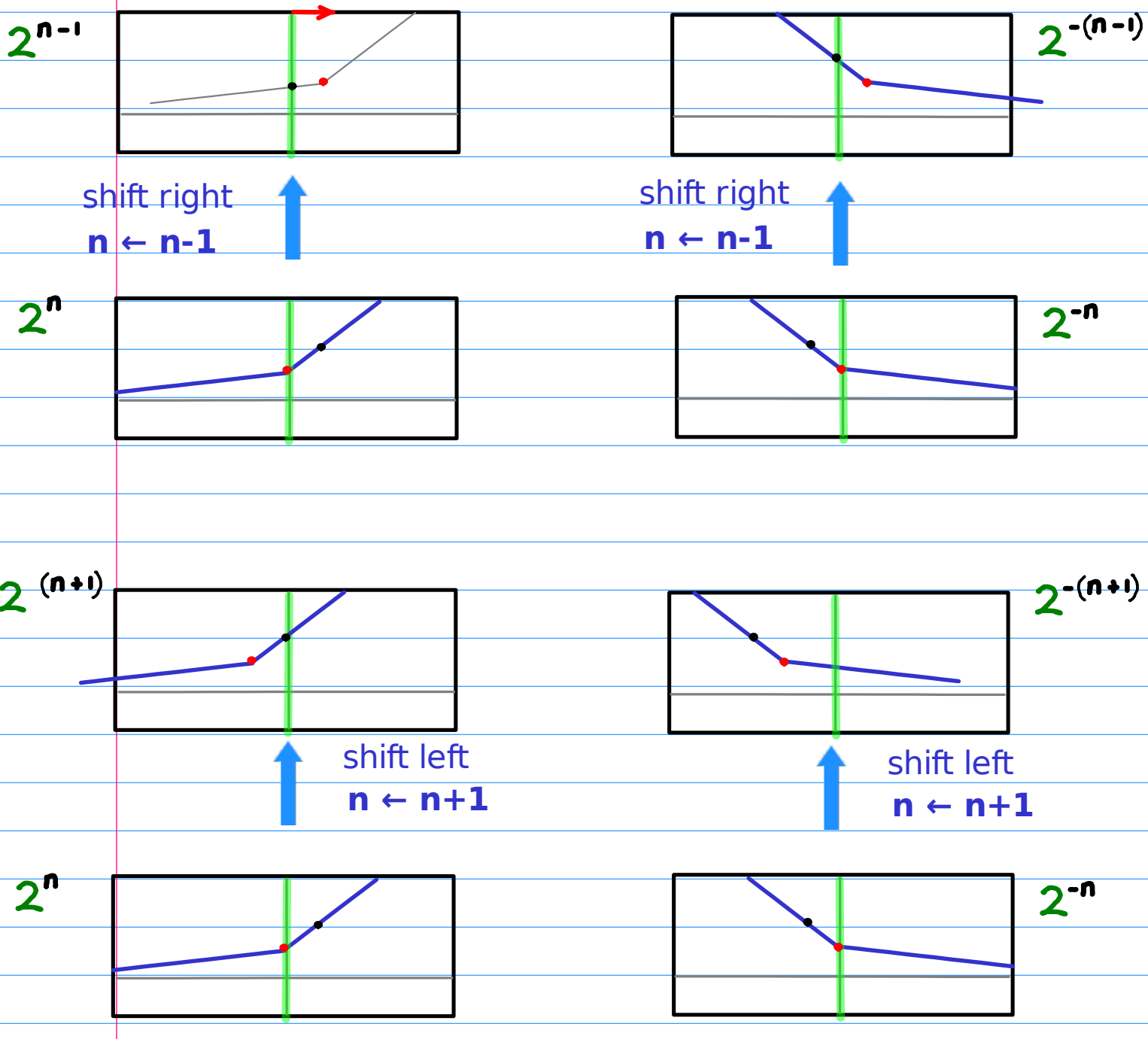
Range Shifting

$$shl(R(n)) = R(n+1)$$
$$shr(R(n)) = R(n-1)$$



Exponent Shifting

$$shl(b^n) = b^{n+1}$$
$$shr(b^n) = b^{n-1}$$

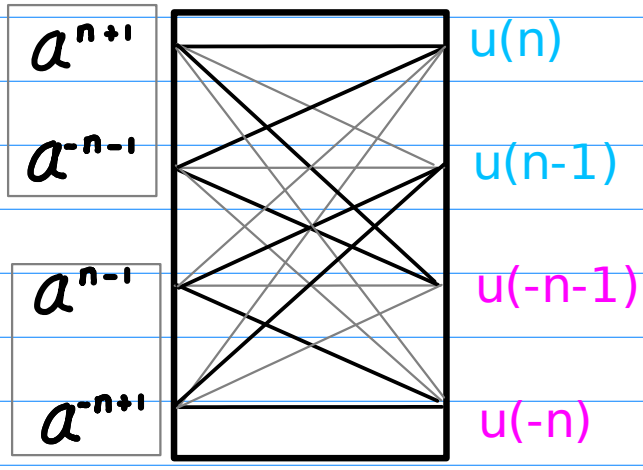


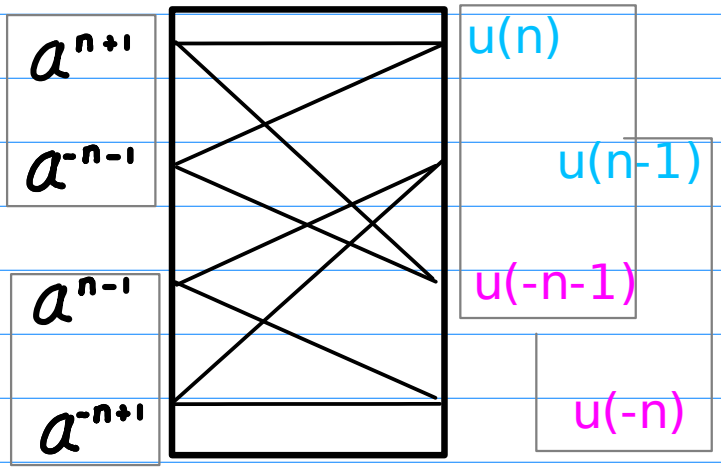
a^n × $R(n)$

a^{n+1}	a^{-n-1}
a^{n-1}	a^{-n+1}

×

u(n)	u(-n-1)
u(n-1)	u(-n)





n ← n+2 or
n ← n - 2

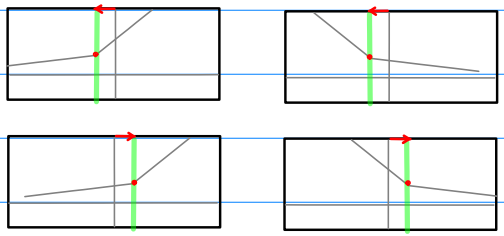
$shl(b^n)$	(1')	a^{n+1}u(n)	a^{-n-1}u(n)	(2')	$shr(b^n)$	(1'')	a^{n-1}u(n)	a^{-n+1}u(n)	(2'')
$shr(b^n)$	(3')	a^{n-1}u(-n)	a^{-n+1}u(-n)	(4')	$shl(b^n)$	(3'')	a^{n+1}u(-n)	a^{-n-1}u(-n)	(4'')
$shl(b^n)$	(5')	a^{n+1}u(-n-1)	a^{-n-1}u(-n-1)	(6')	$shr(b^n)$	(5'')	a^{n-1}u(-n-1)	a^{-n+1}u(-n-1)	(6'')
$shr(b^n)$	(7')	a^{n-1}u(n-1)	a^{-n+1}u(n-1)	(8')	$shl(b^n)$	(7'')	a^{n+1}u(n-1)	a^{-n-1}u(n-1)	(8'')

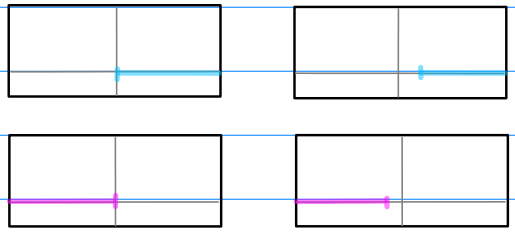
a^n × $R(n)$

a^{n+1}	a^{-n-1}
a^{n-1}	a^{-n+1}

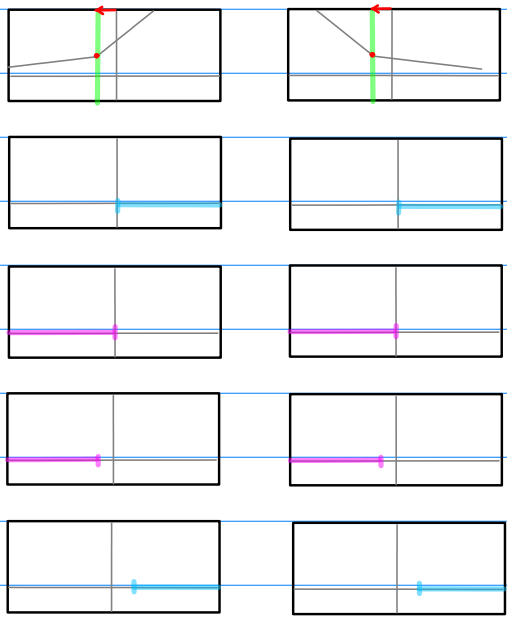
×

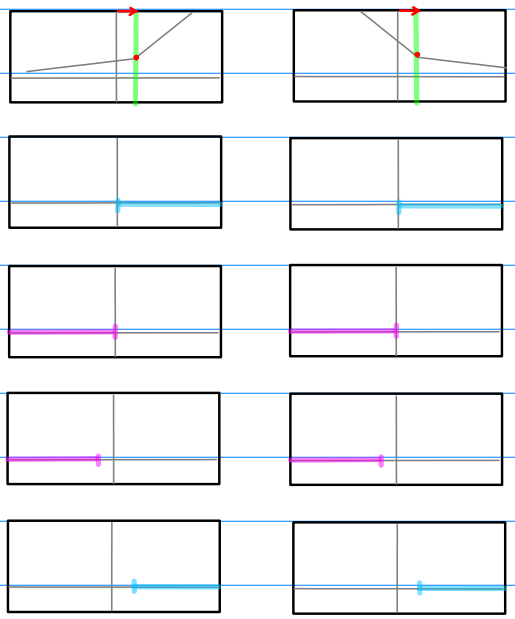
u(n)	u(-n-1)
u(-n)	u(n-1)



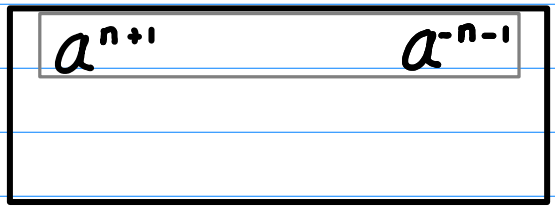


$shl(b^n)$	(1')	a^{n+1}u(n)	a^{-n-1}u(n)	(2')	$shr(b^n)$	(1'')	a^{n-1}u(n)	a^{-n+1}u(n)	(2'')
$shr(b^n)$	(3')	a^{n-1}u(-n)	a^{-n+1}u(-n)	(4')	$shl(b^n)$	(3'')	a^{n+1}u(-n)	a^{-n-1}u(-n)	(4'')
$shl(b^n)$	(5')	a^{n+1}u(-n-1)	a^{-n-1}u(-n-1)	(6')	$shr(b^n)$	(5'')	a^{n-1}u(-n-1)	a^{-n+1}u(-n-1)	(6'')
$shr(b^n)$	(7')	a^{n-1}u(n-1)	a^{-n+1}u(n-1)	(8')	$shl(b^n)$	(7'')	a^{n+1}u(n-1)	a^{-n-1}u(n-1)	(8'')





a^n × $R(n)$



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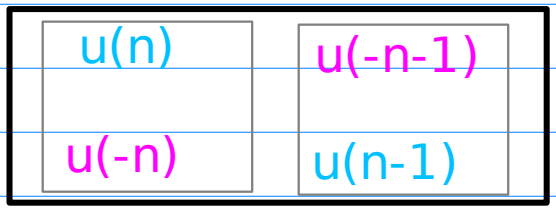
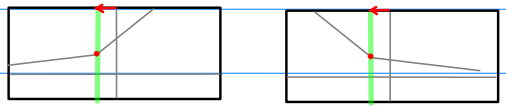
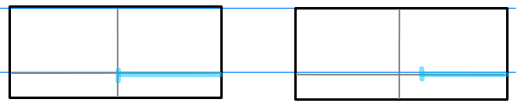
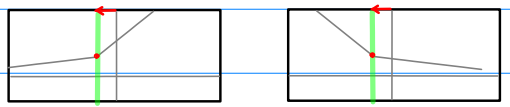
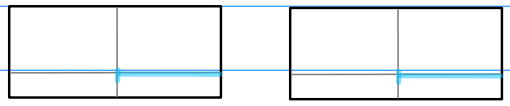


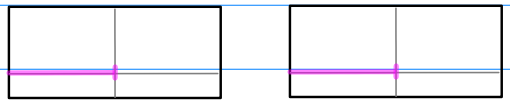


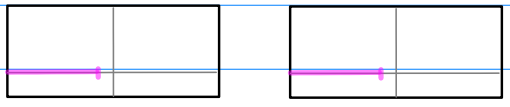


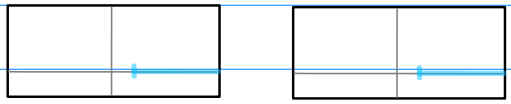
$shl(b^n)$	(1')	a^{n+1}u(n)	a^{-n-1}u(n)	(2')
$shr(b^n)$	(3')	a^{n-1}u(-n)	a^{-n+1}u(-n)	(4')
$shl(b^n)$	(5')	a^{n+1}u(-n-1)	a^{-n-1}u(-n-1)	(6')
$shr(b^n)$	(7')	a^{n-1}u(n-1)	a^{-n+1}u(n-1)	(8')



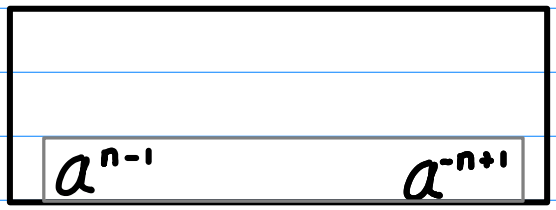








a^n × $R(n)$



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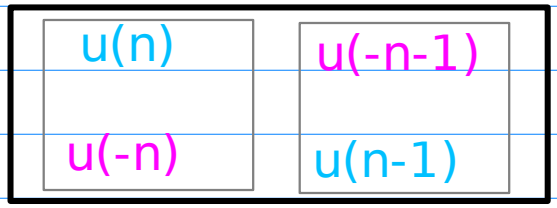
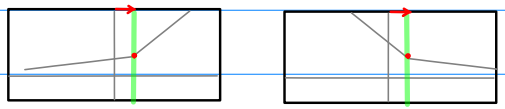
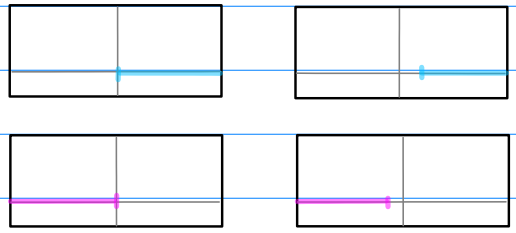
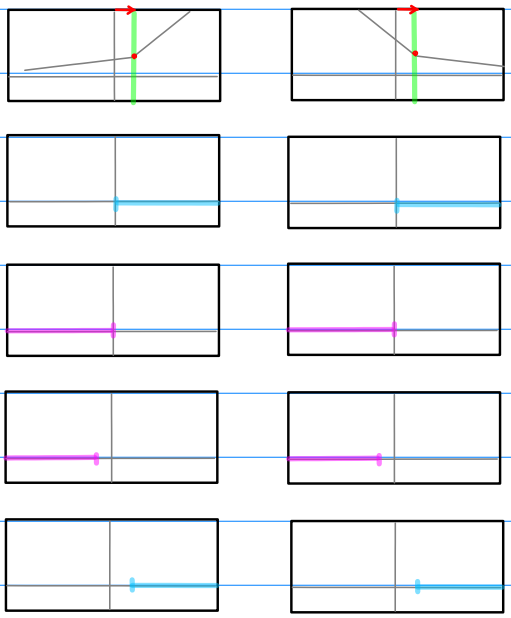


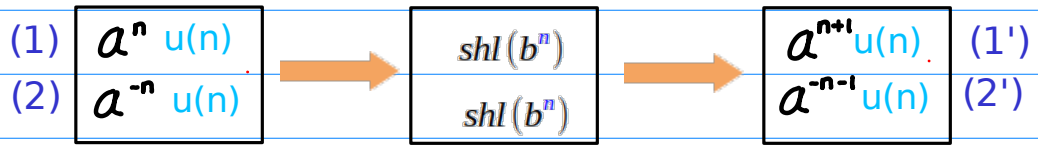


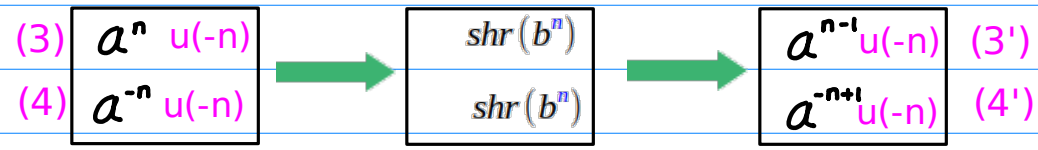


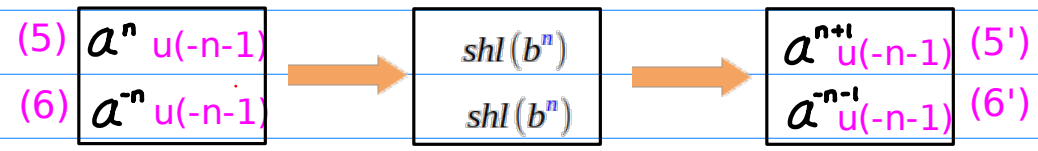
$shr(b^n)$	(1'') a^{n-1} u(n)	a^{-n+1} u(n) (2'')
$shl(b^n)$	(3'') a^{n+1} u(-n)	a^{-n-1} u(-n) (4'')
$shr(b^n)$	(5'') a^{n-1} u(-n-1)	a^{-n+1} u(-n-1) (6'')
$shl(b^n)$	(7'') a^{n+1} u(n-1)	a^{-n-1} u(n-1) (8'')

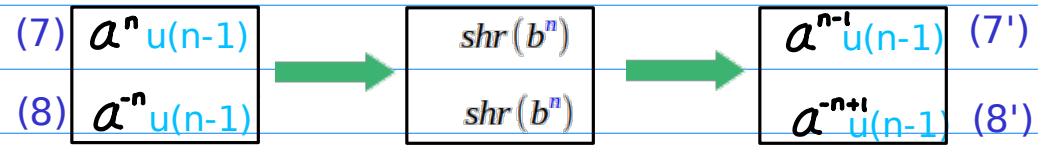


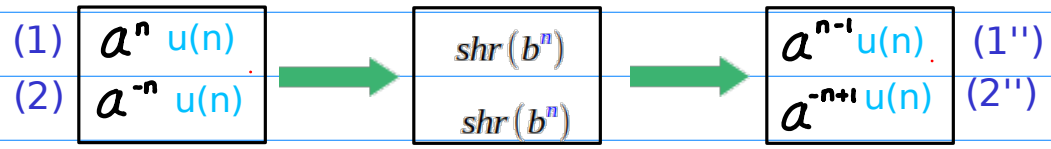
Shifted Sequence 1 x' **Shifted Sequence 2 x''**

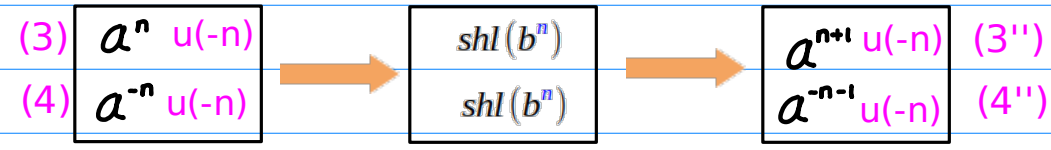


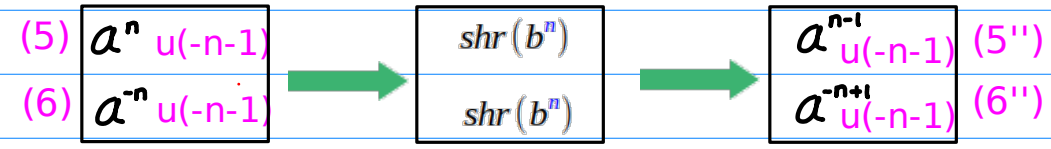


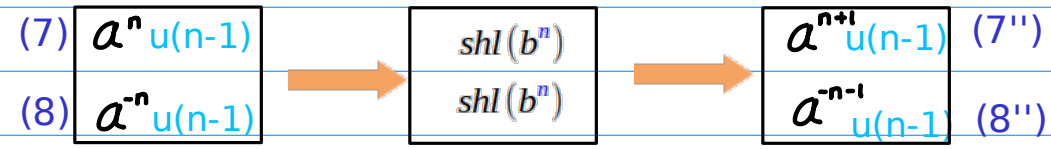










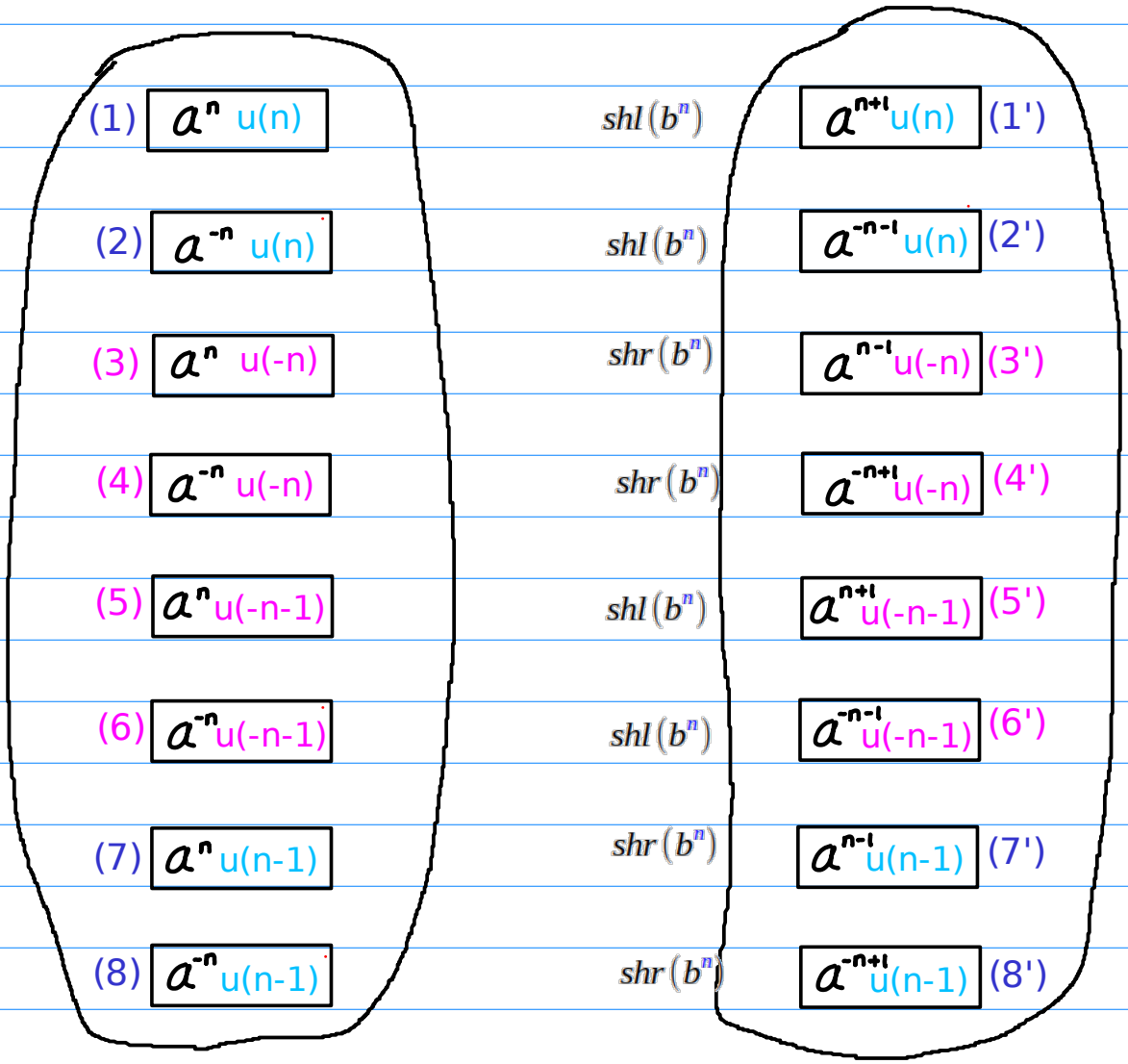


Unshifted Sequence x		Shifted Sequence 1 x'		Shifted Sequence 2 x''	
(1)	$a^n\ u(n)$	$shl(b^n)$	$a^{n+1}u(n)$ (1')	$shr(b^n)$	$a^{n-1}\ u(n)$ (1'')
(2)	$a^{-n}\ u(n)$	$shl(b^n)$	$a^{-n-1}u(n)$ (2')	$shr(b^n)$	$a^{-n+1}\ u(n)$ (2'')
(3)	$a^n\ u(-n)$	$shr(b^n)$	$a^{n-1}u(-n)$ (3')	$shl(b^n)$	$a^{n+1}\ u(-n)$ (3'')
(4)	$a^{-n}\ u(-n)$	$shr(b^n)$	$a^{-n+1}u(-n)$ (4')	$shl(b^n)$	$a^{-n-1}u(-n)$ (4'')
(5)	$a^n u(-n-1)$	$shl(b^n)$	$a^{n+1}u(-n-1)$ (5')	$shr(b^n)$	$a^{n-1}u(-n-1)$ (5'')
(6)	$a^{-n}u(-n-1)$	$shl(b^n)$	$a^{-n-1}u(-n-1)$ (6')	$shr(b^n)$	$a^{-n+1}u(-n-1)$ (6'')
(7)	$a^n u(n-1)$	$shr(b^n)$	$a^{n-1}u(n-1)$ (7')	$shl(b^n)$	$a^{n+1}u(n-1)$ (7'')
(8)	$a^{-n}u(n-1)$	$shr(b^n)$	$a^{-n+1}u(n-1)$ (8')	$shl(b^n)$	$a^{-n-1}u(n-1)$ (8'')

$shl(b^n) \Longleftrightarrow shr(b^n)$

$shr(b^n) \Longleftrightarrow shl(b^n)$

many possible permuations are possible
but consider these two

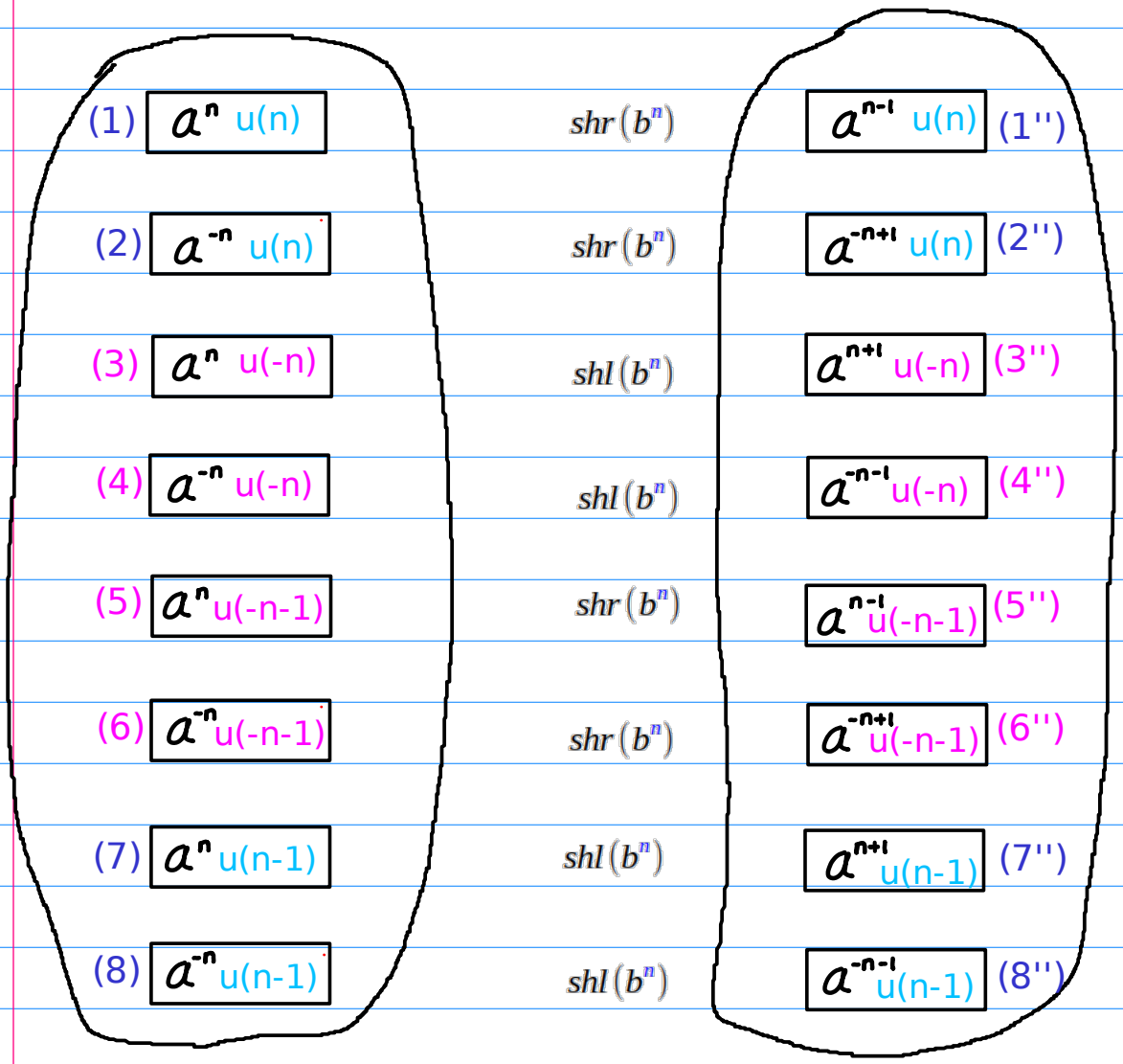


Inter-permutations over unshifted sequnce and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence

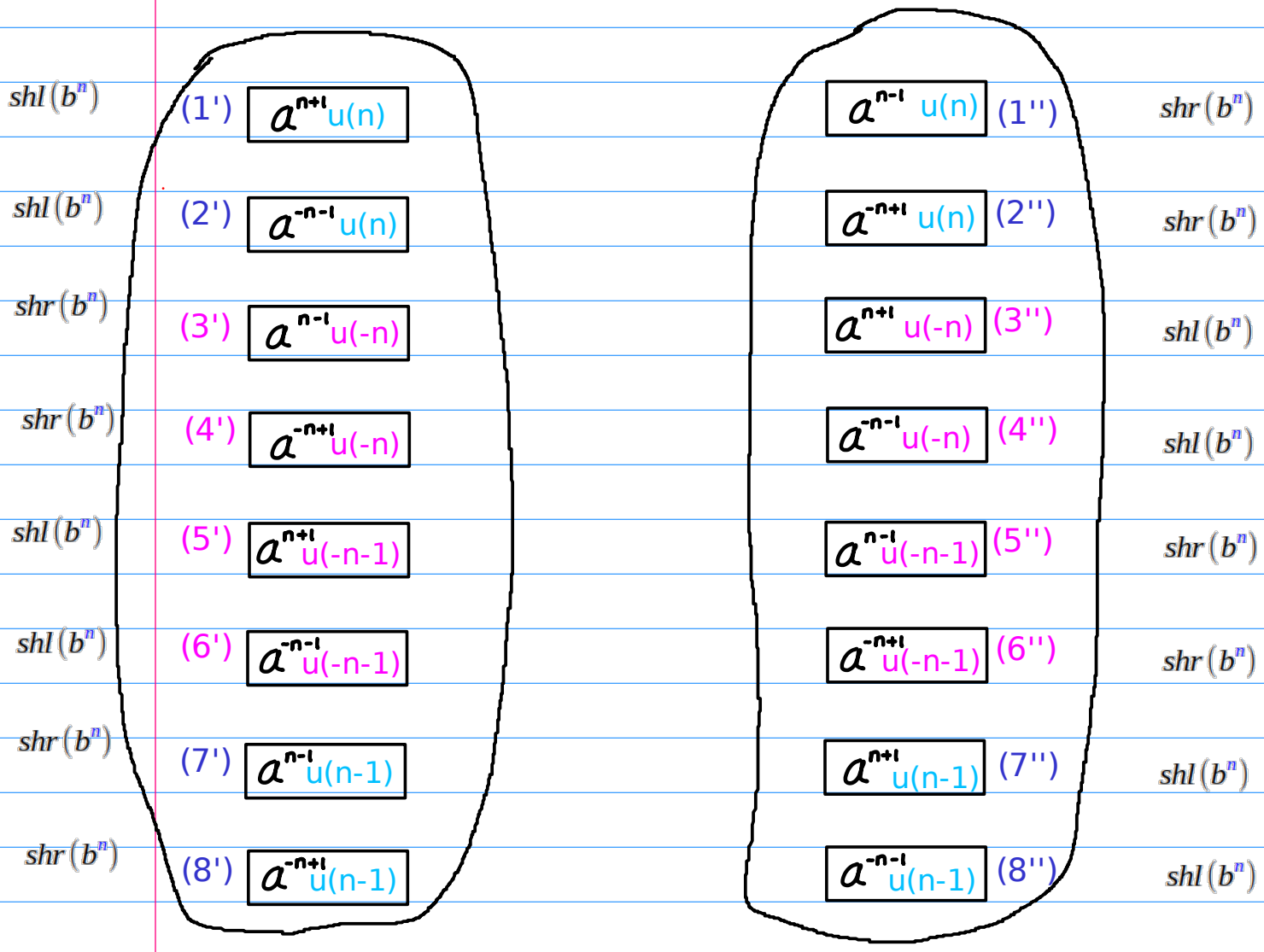
Unshifted Sequence x → **Shifted Sequence 2 x''**



Inter-permutations over unshifted sequnce and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence



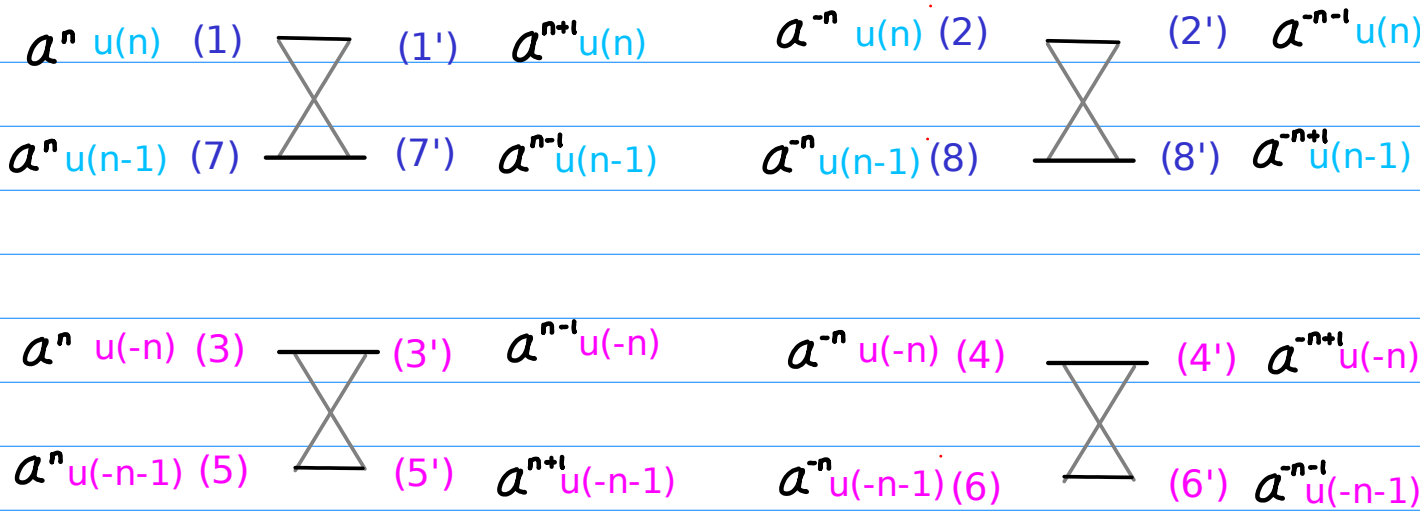
Inter-permutations over unshifted sequnce and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence

A.1 (x) → (x')

(1)~(8) (1')~(8')



Horizontal Exponent Conversion

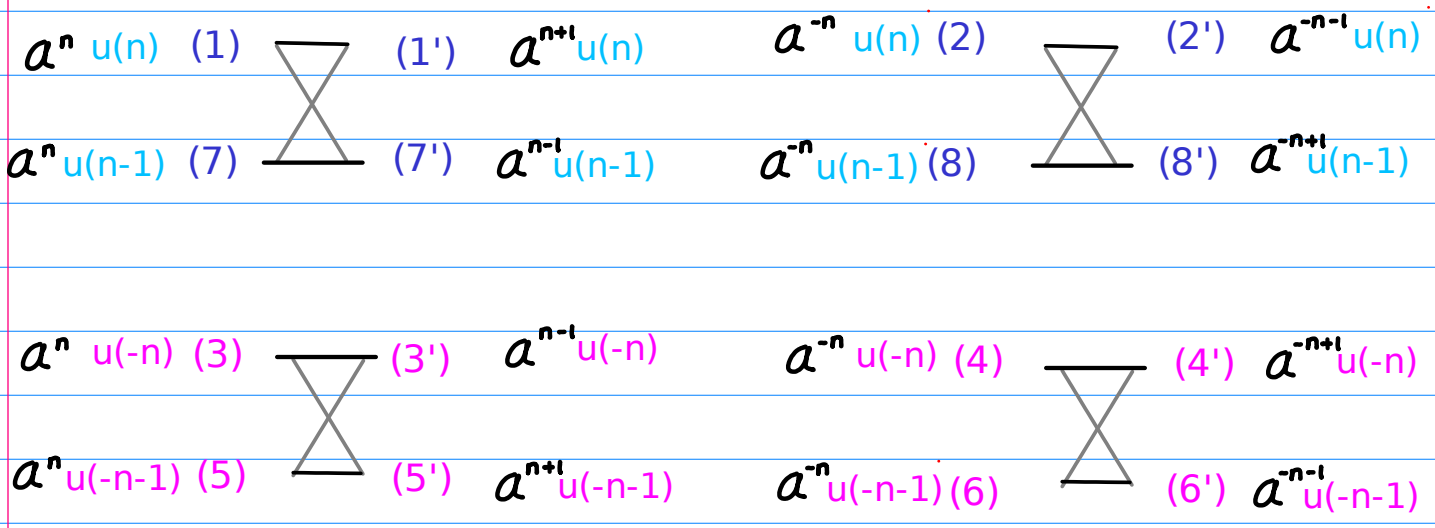
(1) n	(1') (n+1)	shl(n)
(2) -n	(2') -(n+1)	shl(n)
(3) n	(3') (n−1)	shr(n)
(4) -n	(4') -(n−1)	shr(n)
(5) n	(5') (n+1)	shl(n)
(6) -n	(6') -(n+1)	shl(n)
(7) n	(7') (n−1)	shr(n)
(8) -n	(8') -(n−1)	shr(n)

Diagonal Exponent Conversion

(1) n	(1') (n+1)	shl(n)
(2) -n	(2') -(n+1)	shl(n)
(3) n	(3') (n−1)	shr(n)
(4) -n	(4') -(n−1)	shr(n)
(5) n	(5') (n+1)	shl(n)
(6) -n	(6') -(n+1)	shl(n)
(7) n	(7') (n−1)	shr(n)
(8) -n	(8') -(n−1)	shr(n)

A.2 (x) → (x')

(1)~(8) (1')~(8')



Horizontal Range Conversion

(1) u(n)	(1') u(n)	id(n)
(2) u(n)	(2') u(n)	id(n)
(3) u(-n)	(3') u(-n)	id(n)
(4) u(-n)	(4') u(-n)	id(n)
(5) u(-n-1)	(5') u(-n-1)	id(n)
(6) u(-n-1)	(6') u(-n-1)	id(n)
(7) u(n-1)	(7') u(n-1)	id(n)
(8) u(n-1)	(8') u(n-1)	id(n)

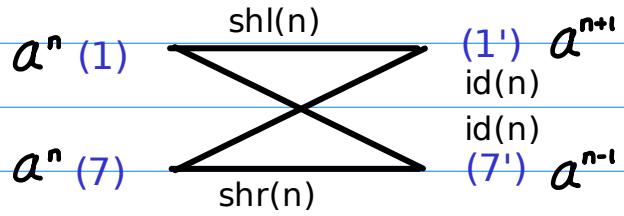
Diagonal Range Conversion

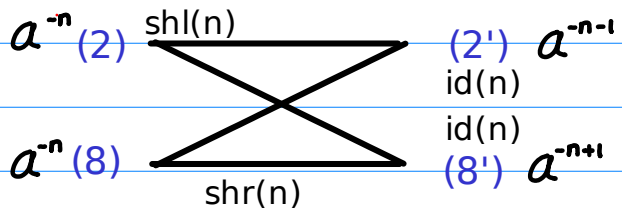
(1) u(n)	(1') u(n)	shl(n)
(2) u(n)	(2') u(n)	shl(n)
(3) u(-n)	(3') u(-n)	shr(n)
(4) u(-n)	(4') u(-n)	shr(n)
(5) u(-n-1)	(5') u(-n-1)	shl(n)
(6) u(-n-1)	(6') u(-n-1)	shl(n)
(7) u(n-1)	(7') u(n-1)	shr(n)
(8) u(n-1)	(8') u(n-1)	shr(n)

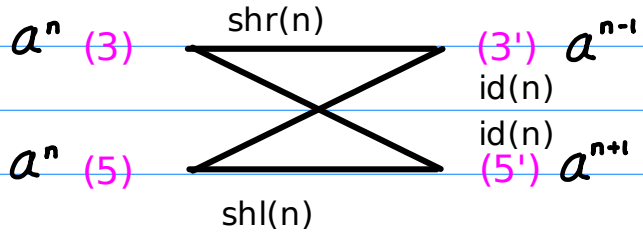
A.3 (x) → (x')

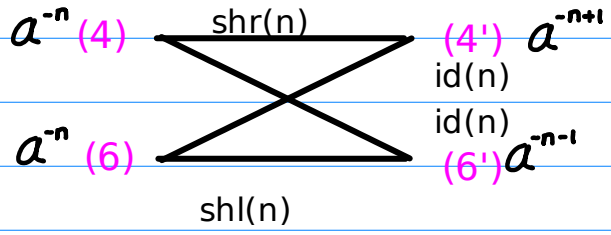
(1)~(8) (1')~(8')

Exponent Conversion

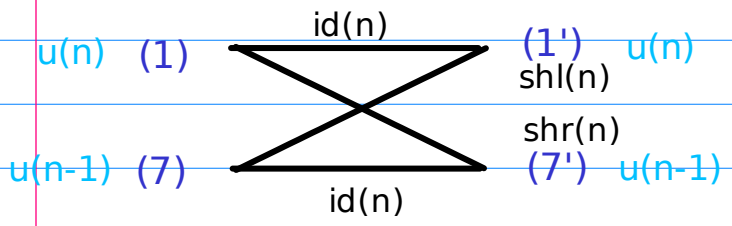


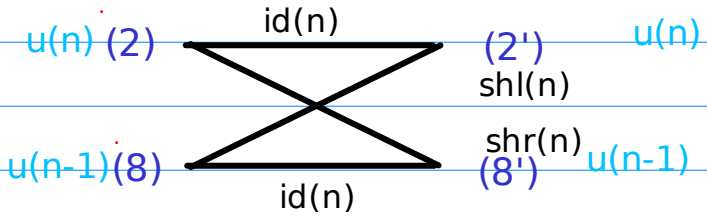


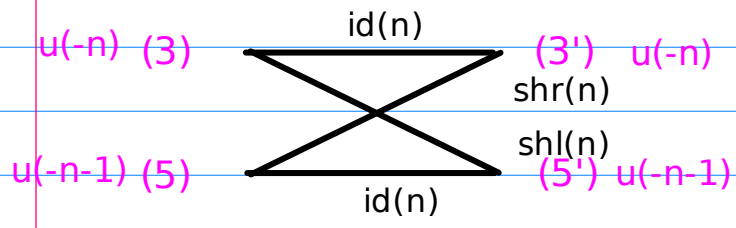


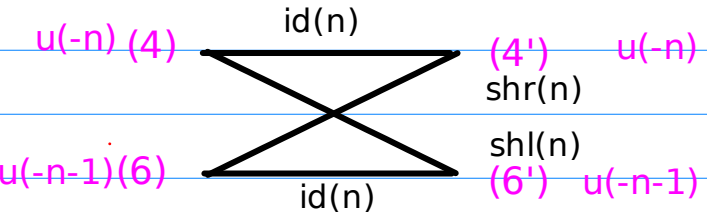


Range Conversion





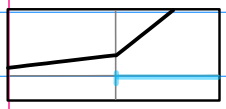




A.4 (x) → (x')

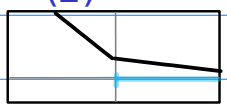
(1)~(8) (1')~(8')

(1)



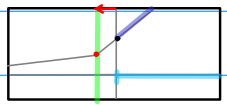
a^n u(n)

(2)



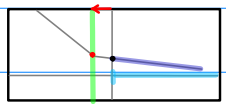
a^{-n} u(n)

(1')



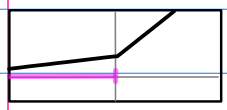
a^{n+1} u(n)

(2')



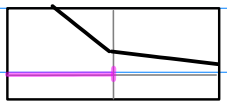
a^{-n-1} u(n)

(3)



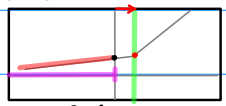
a^n u(-n)

(4)



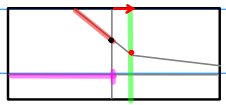
a^{-n} u(-n)

(3')



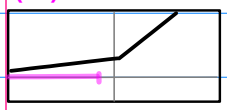
a^{n-1} u(-n)

(4')



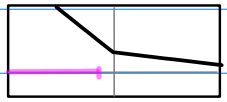
a^{-n+1} u(-n)

(5)



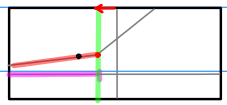
a^n u(-n-1)

(6)



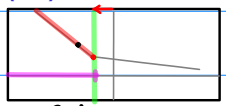
a^{-n} u(-n-1)

(5')



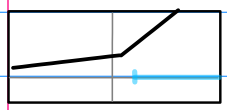
a^{n+1} u(-n-1)

(6')



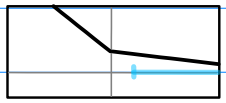
a^{-n-1} u(-n-1)

(7)



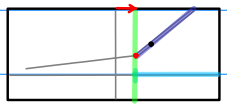
a^n u(n-1)

(8)



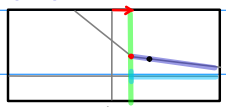
a^{-n} u(n-1)

(7')



a^{n-1} u(n-1)

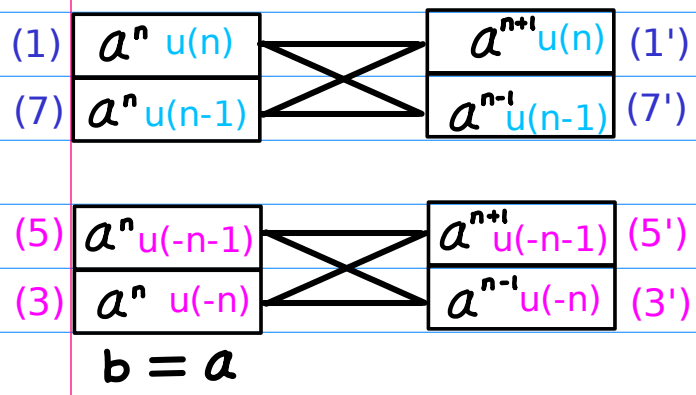
(8')

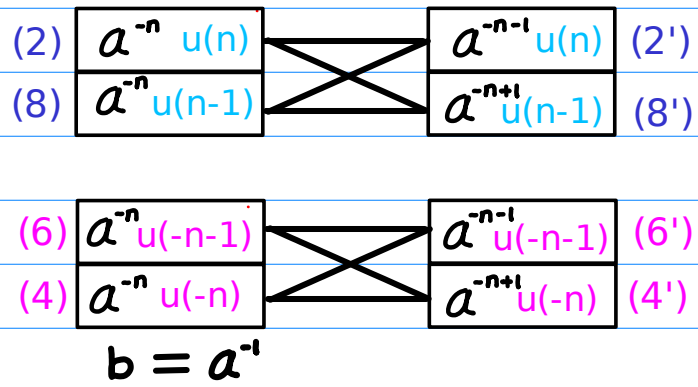


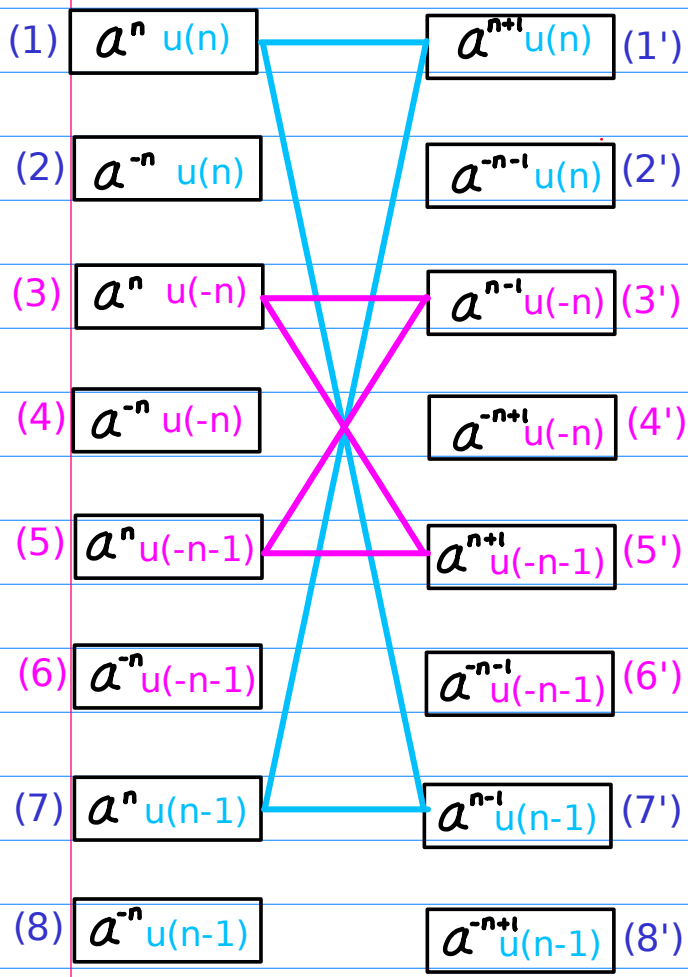
a^{-n+1} u(n-1)

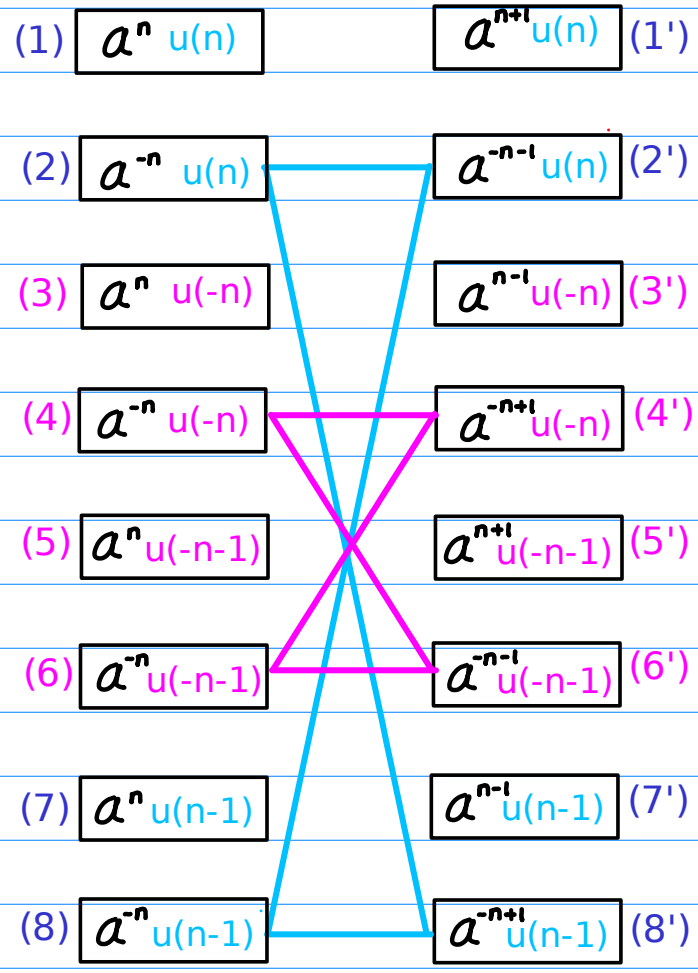
A.5 (x) → (x')

(1)~(8) (1')~(8')


(1) a^n u(n)
(7) a^n u(n-1)
(5) a^n u(-n-1)
(3) a^n u(-n)
a^{n+1} u(n) (1')
a^{n-1} u(n-1) (7')
a^{n+1} u(-n-1) (5')
a^{n-1} u(-n) (3')
b = a

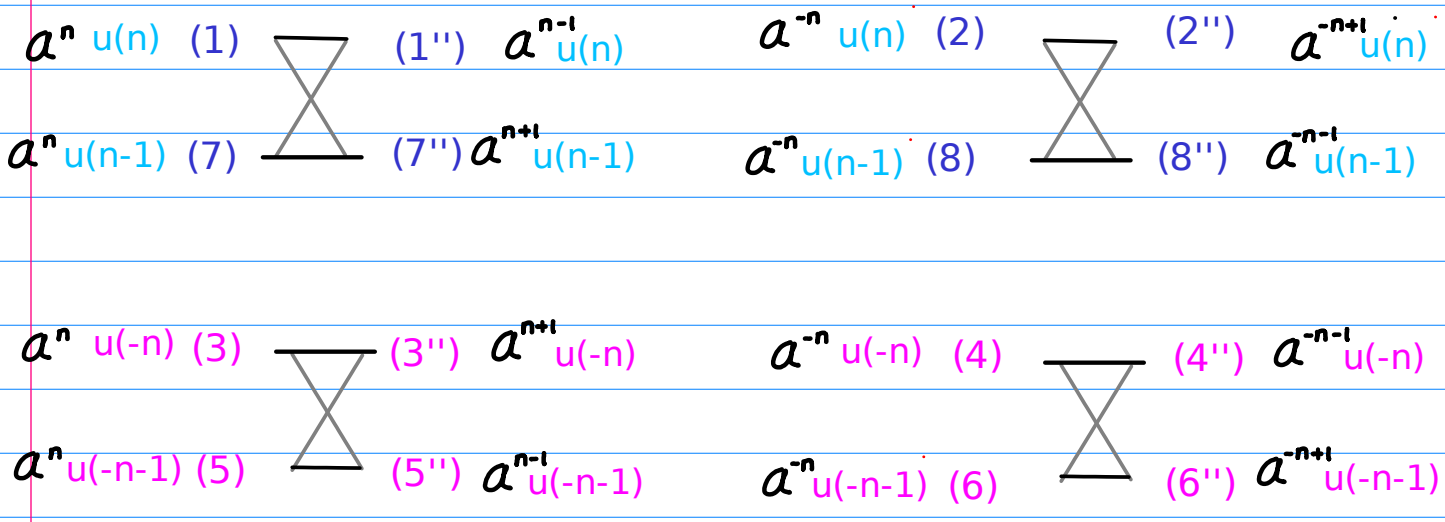

(2) a^{-n} u(n)
(8) a^{-n} u(n-1)
(6) a^{-n} u(-n-1)
(4) a^{-n} u(-n)
a^{-n-1} u(n) (2')
a^{-n+1} u(n-1) (8')
a^{-n-1} u(-n-1) (6')
a^{-n+1} u(-n) (4')
b = a^{-1}


(1) a^n u(n)
(2) a^{-n} u(n)
(3) a^n u(-n)
(4) a^{-n} u(-n)
(5) a^n u(-n-1)
(6) a^{-n} u(-n-1)
(7) a^n u(n-1)
(8) a^{-n} u(n-1)
a^{n+1} u(n) (1')
a^{-n-1} u(n) (2')
a^{n-1} u(-n) (3')
a^{-n+1} u(-n) (4')
a^{n+1} u(-n-1) (5')
a^{-n-1} u(-n-1) (6')
a^{n-1} u(n-1) (7')
a^{-n+1} u(n-1) (8')


(1) a^n u(n)
(2) a^{-n} u(n)
(3) a^n u(-n)
(4) a^{-n} u(-n)
(5) a^n u(-n-1)
(6) a^{-n} u(-n-1)
(7) a^n u(n-1)
(8) a^{-n} u(n-1)
a^{n+1} u(n) (1')
a^{-n-1} u(n) (2')
a^{n-1} u(-n) (3')
a^{-n+1} u(-n) (4')
a^{n+1} u(-n-1) (5')
a^{-n-1} u(-n-1) (6')
a^{n-1} u(n-1) (7')
a^{-n+1} u(n-1) (8')

B.1 (x) → (x'')

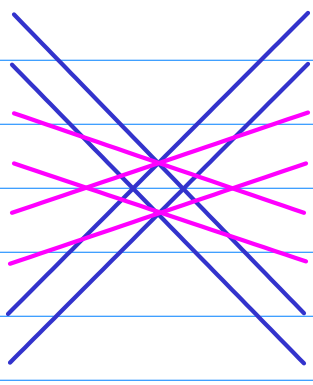
(1)~(8) (1'')~(8'')



Horizontal Exponent Conversion

(1)	n	(1'') (n−1)	shr(n)
(2)	-n	(2'') -(n−1)	shr(n)
(3)	n	(3'') (n+1)	shl(n)
(4)	-n	(4'') -(n+1)	shl(n)
(5)	n	(5'') (n−1)	shr(n)
(6)	-n	(6'') -(n−1)	shr(n)
(7)	n	(7'') (n+1)	shl(n)
(8)	-n	(8'') -(n+1)	shl(n)

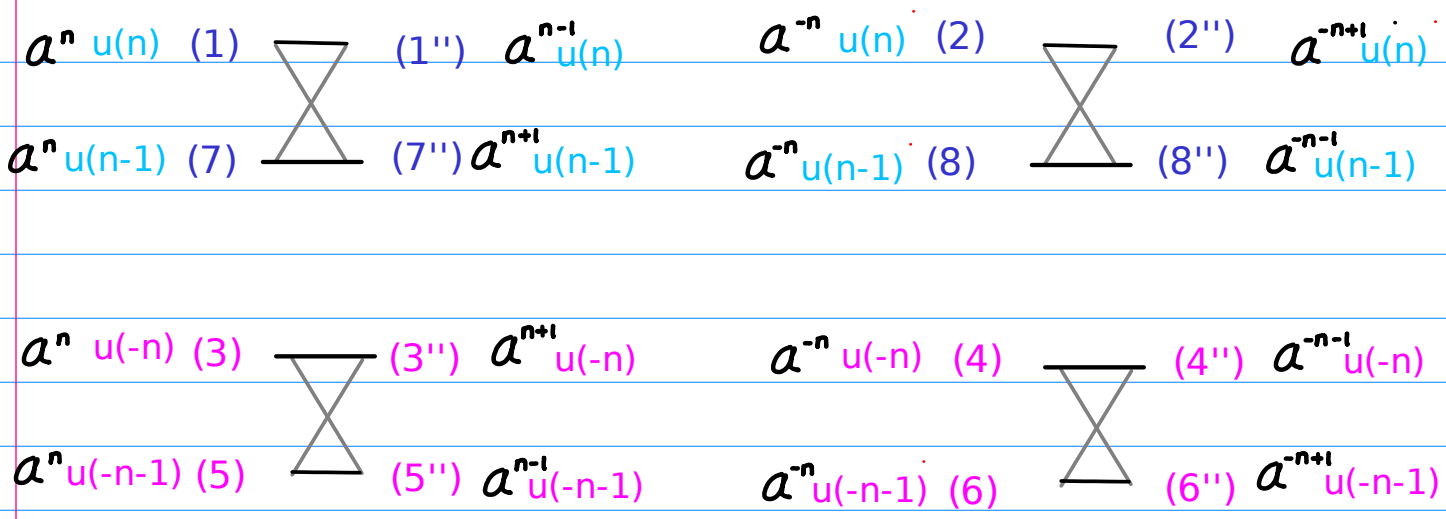
Diagonal Exponent Conversion



(1)	n	(1'') (n+1)	shl(n)
(2)	-n	(2'') -(n+1)	shl(n)
(3)	n	(3'') (n−1)	shr(n)
(4)	-n	(4'') -(n−1)	shr(n)
(5)	n	(5'') (n+1)	shl(n)
(6)	-n	(6'') -(n+1)	shl(n)
(7)	n	(7'') (n−1)	shr(n)
(8)	-n	(8'') -(n−1)	shr(n)

B.2 (x) → (x'')

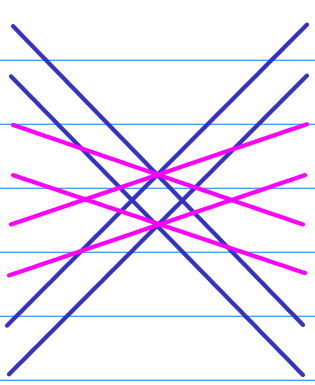
(1)~(8) (1'')~(8'')



Horizontal Range Conversion

(1) u(n)	———	(1'') u(n)	id(n)
(2) u(n)	———	(2'') u(n)	id(n)
(3) u(-n)	———	(3'') u(-n)	id(n)
(4) u(-n)	———	(4'') u(-n)	id(n)
(5) u(-n-1)	———	(5'') u(-n-1)	id(n)
(6) u(-n-1)	———	(6'') u(-n-1)	id(n)
(7) u(n-1)	———	(7'') u(n-1)	id(n)
(8) u(n-1)	———	(8'') u(n-1)	id(n)

Diagonal Range Conversion



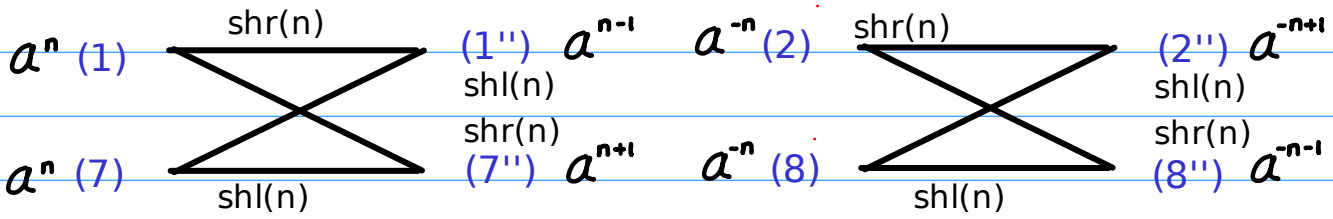
(1) u(n)	(1'') u(n)	shl(n)
(2) u(n)	(2'') u(n)	shl(n)
(3) u(-n)	(3'') u(-n)	shr(n)
(4) u(-n)	(4'') u(-n)	shr(n)
(5) u(-n-1)	(5'') u(-n-1)	shl(n)
(6) u(-n-1)	(6'') u(-n-1)	shl(n)
(7) u(n-1)	(7'') u(n-1)	shr(n)
(8) u(n-1)	(8'') u(n-1)	shr(n)

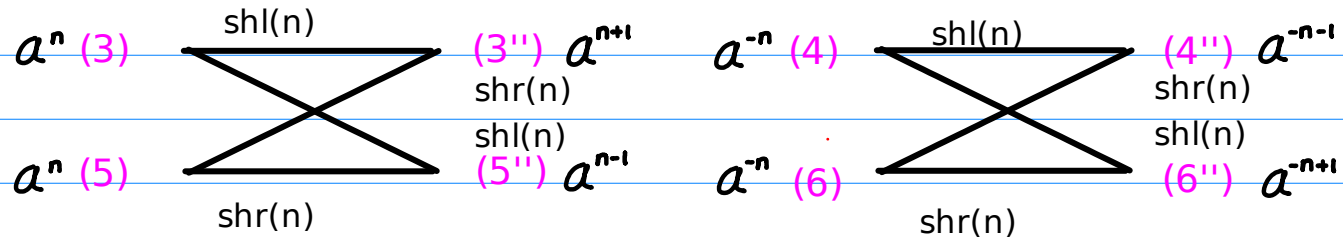
B.3

(x) → **(x'')**

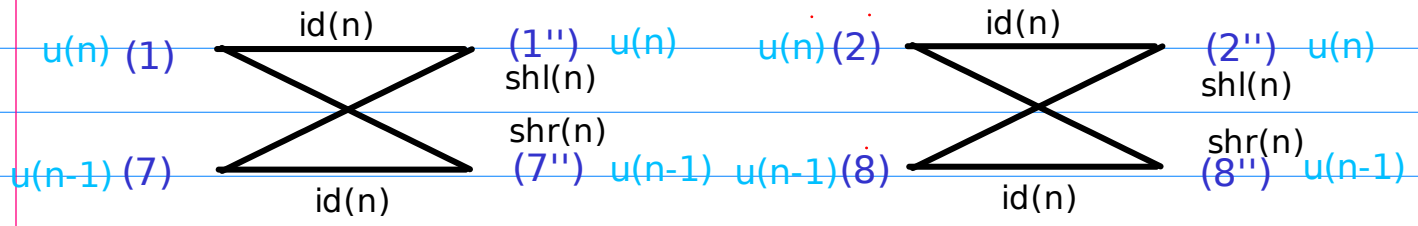
(1)~(8) (1'')~(8'')

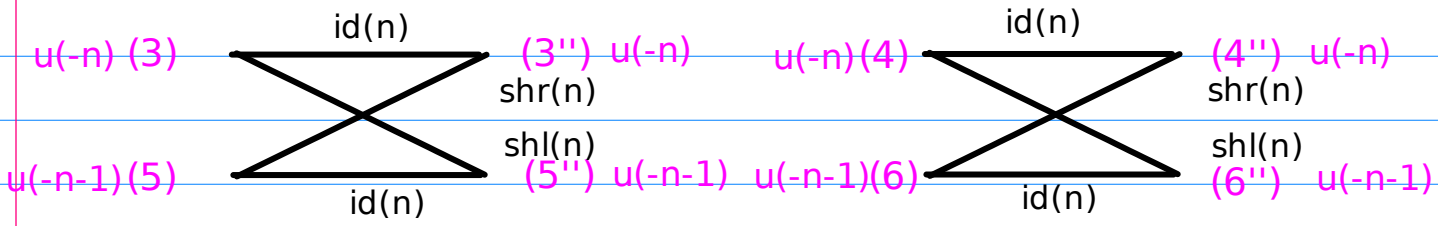
Exponent Conversion





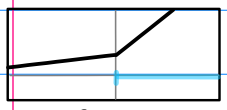
Range Conversion

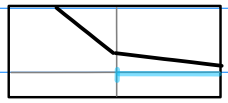


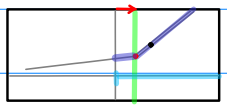


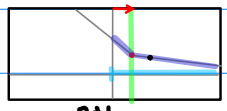
B.4 (x) → (x')

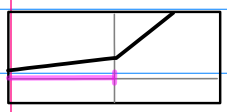
(1)~(8) (1')~(8')

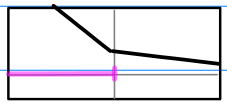
(1)  a^n u(n)

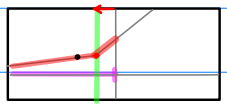
(2)  a^{-n} u(n)

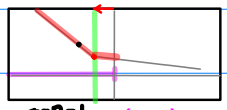
(1'')  a^{n-1}u(n)

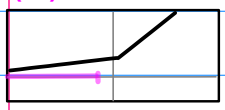
(2'')  a^{-n+1}u(n)

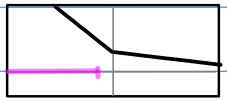
(3)  a^n u(-n)

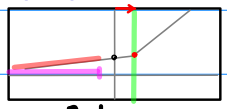
(4)  a^{-n} u(-n)

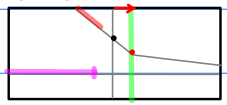
(3'')  a^{n+1} u(-n)

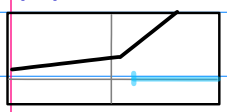
(4'')  a^{-n-1} u(-n)

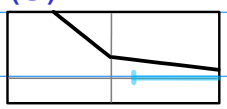
(5)  a^n u(-n-1)

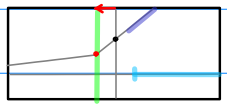
(6)  a^{-n}u(-n-1)

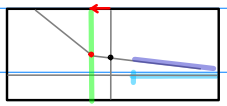
(5'')  a^{n-1} u(-n-1)

(6'')  a^{-n+1}u(-n-1)

(7)  a^n u(n-1)

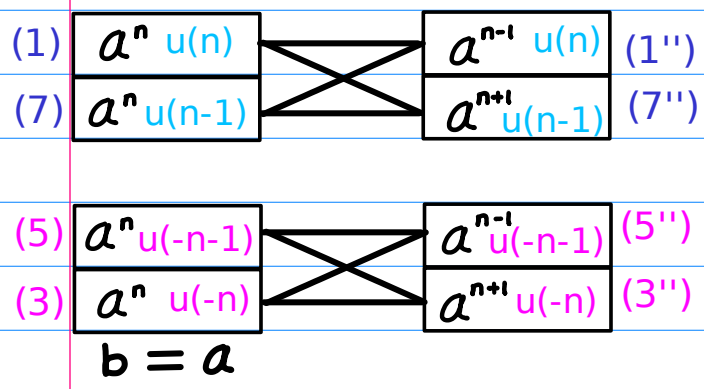
(8)  a^{-n}u(n-1)

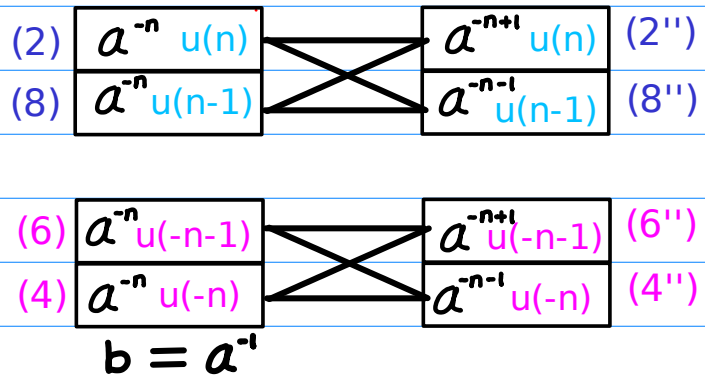
(7'')  a^{n+1} u(n-1)

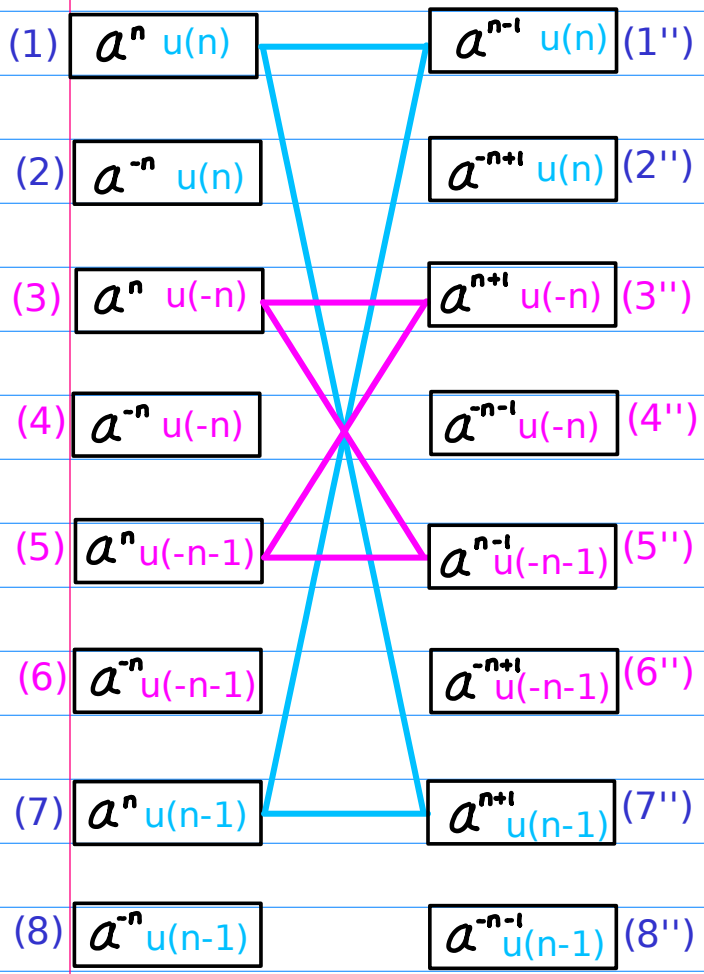
(8'')  a^{-n-1} u(n-1)

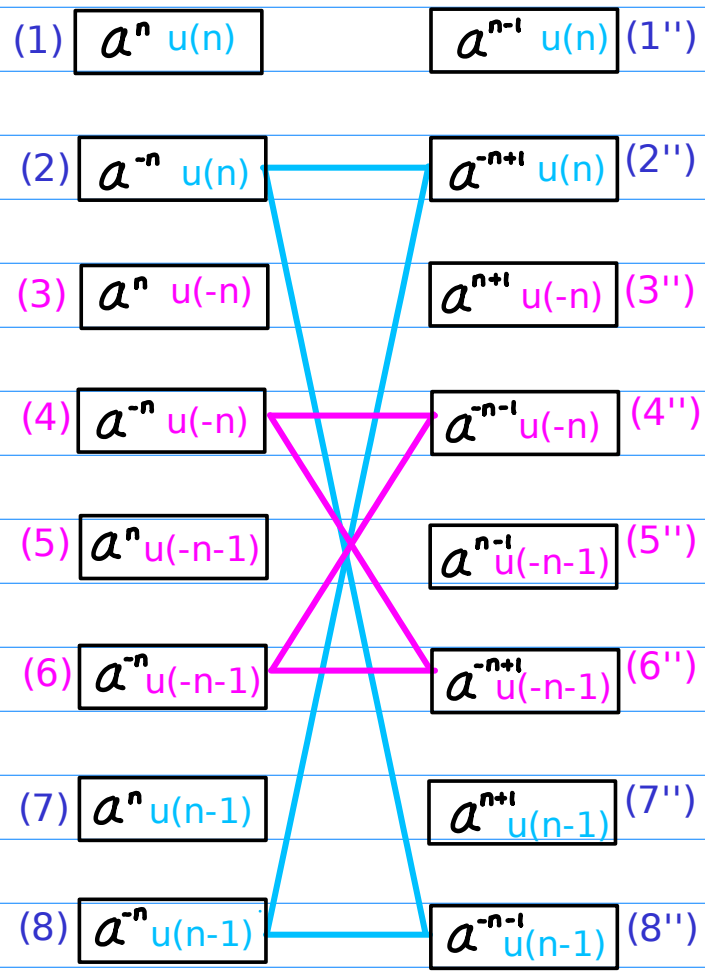
B.5 (x) → (x')

(1)~(8) (1')~(8')


(1) a^n u(n)
(7) a^n u(n-1)
a^{n-l} u(n) (1'')
a^{n+l} u(n-1) (7'')
(5) a^n u(-n-1)
(3) a^n u(-n)
a^{n-l} u(-n-1) (5'')
a^{n+l} u(-n) (3'')
b = a

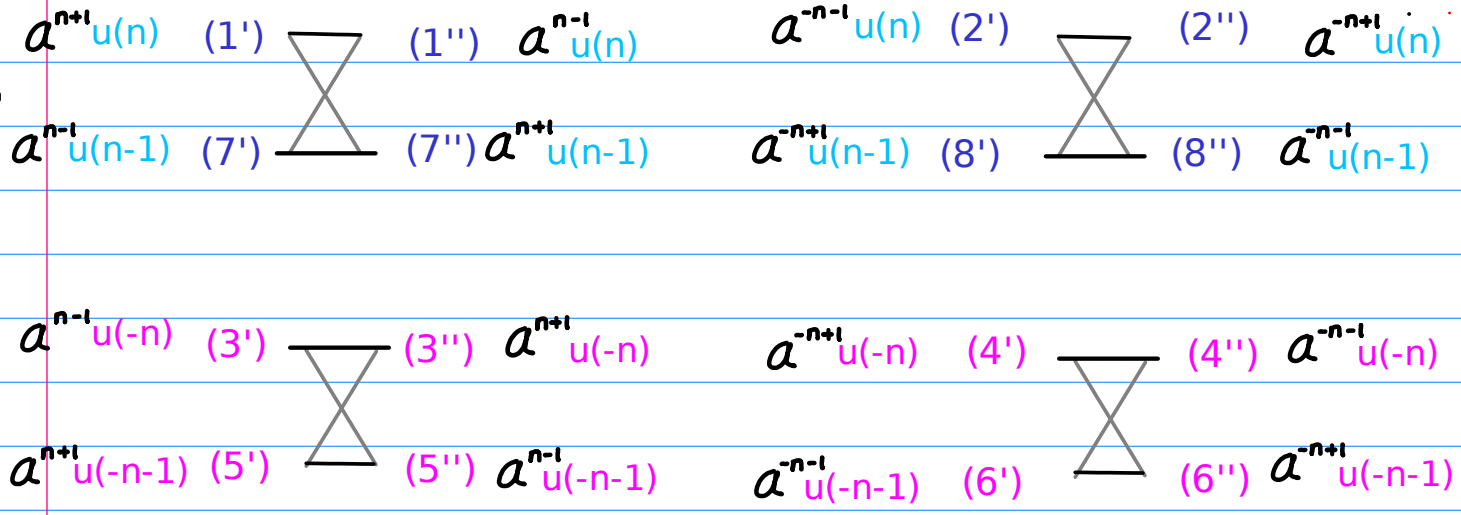

(2) a^{-n} u(n)
(8) a^{-n} u(n-1)
a^{-n+l} u(n) (2'')
a^{-n-l} u(n-1) (8'')
(6) a^{-n} u(-n-1)
(4) a^{-n} u(-n)
a^{-n+l} u(-n-1) (6'')
a^{-n-l} u(-n) (4'')
b = a^{-1}


(1) a^n u(n)
(2) a^{-n} u(n)
(3) a^n u(-n)
(4) a^{-n} u(-n)
(5) a^n u(-n-1)
(6) a^{-n} u(-n-1)
(7) a^n u(n-1)
(8) a^{-n} u(n-1)
a^{n-l} u(n) (1'')
a^{-n+l} u(n) (2'')
a^{n+l} u(-n) (3'')
a^{-n-l} u(-n) (4'')
a^{n-l} u(-n-1) (5'')
a^{-n+l} u(-n-1) (6'')
a^{n+l} u(n-1) (7'')
a^{-n-l} u(n-1) (8'')


(1) a^n u(n)
(2) a^{-n} u(n)
(3) a^n u(-n)
(4) a^{-n} u(-n)
(5) a^n u(-n-1)
(6) a^{-n} u(-n-1)
(7) a^n u(n-1)
(8) a^{-n} u(n-1)
a^{n-l} u(n) (1'')
a^{-n+l} u(n) (2'')
a^{n+l} u(-n) (3'')
a^{-n-l} u(-n) (4'')
a^{n-l} u(-n-1) (5'')
a^{-n+l} u(-n-1) (6'')
a^{n+l} u(n-1) (7'')
a^{-n-l} u(n-1) (8'')

C.1 (x') → (x'')

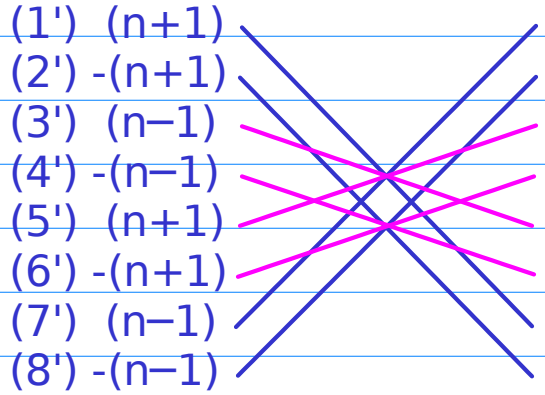
(1')~(8') (1'')~(8'')



Horizontal Exponent Conversion

(1') (n+1)	(1'') (n−1)	shr(shr(n))
(2') -(n+1)	(2'') -(n−1)	shr(shr(n))
(3') (n−1)	(3'') (n+1)	shr(shr(n))
(4') -(n−1)	(4'') -(n+1)	shr(shr(n))
(5') (n+1)	(5'') (n−1)	shl(shl(n))
(6') -(n+1)	(6'') -(n−1)	shl(shl(n))
(7') (n−1)	(7'') (n+1)	shl(shl(n))
(8') -(n−1)	(8'') -(n+1)	shl(shl(n))

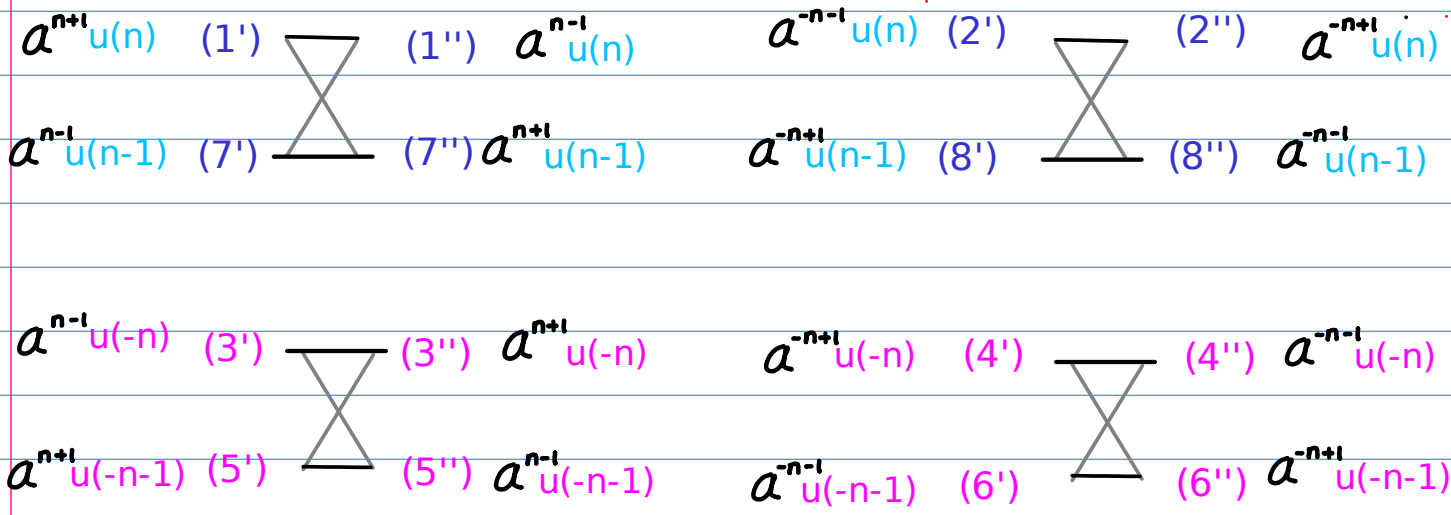
Diagonal Exponent Conversion



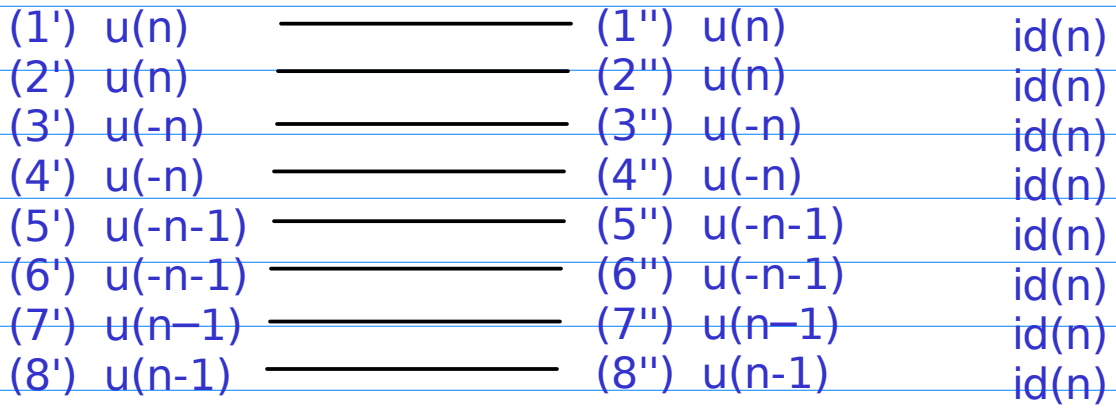
id(n)
id(n)
id(n)
id(n)
id(n)
id(n)
id(n)
id(n)

C.2 (x') → (x'')

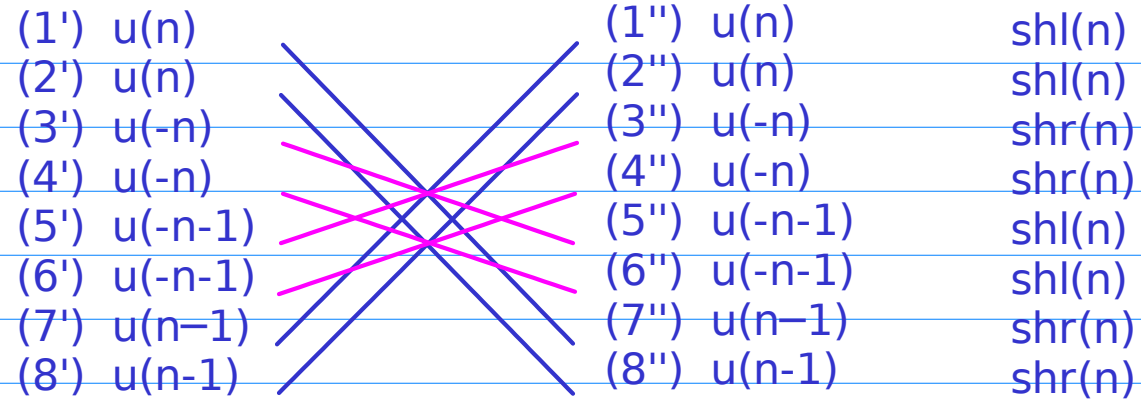
(1')~(8') (1'')~(8'')



Horizontal Range Conversion



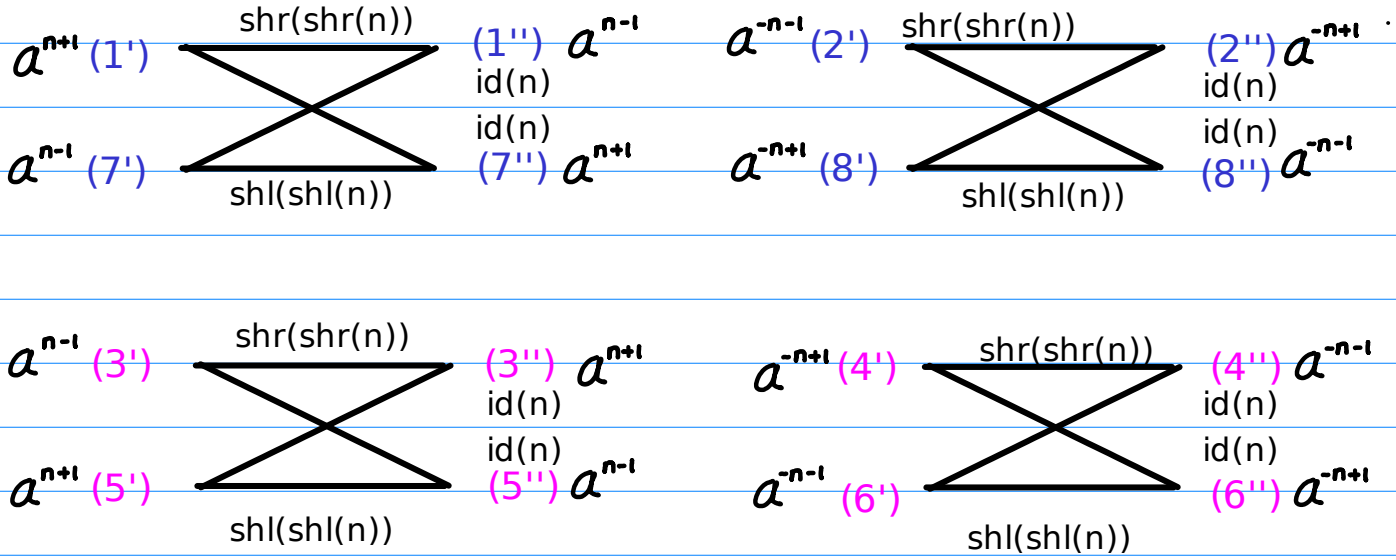
Diagonal Range Conversion



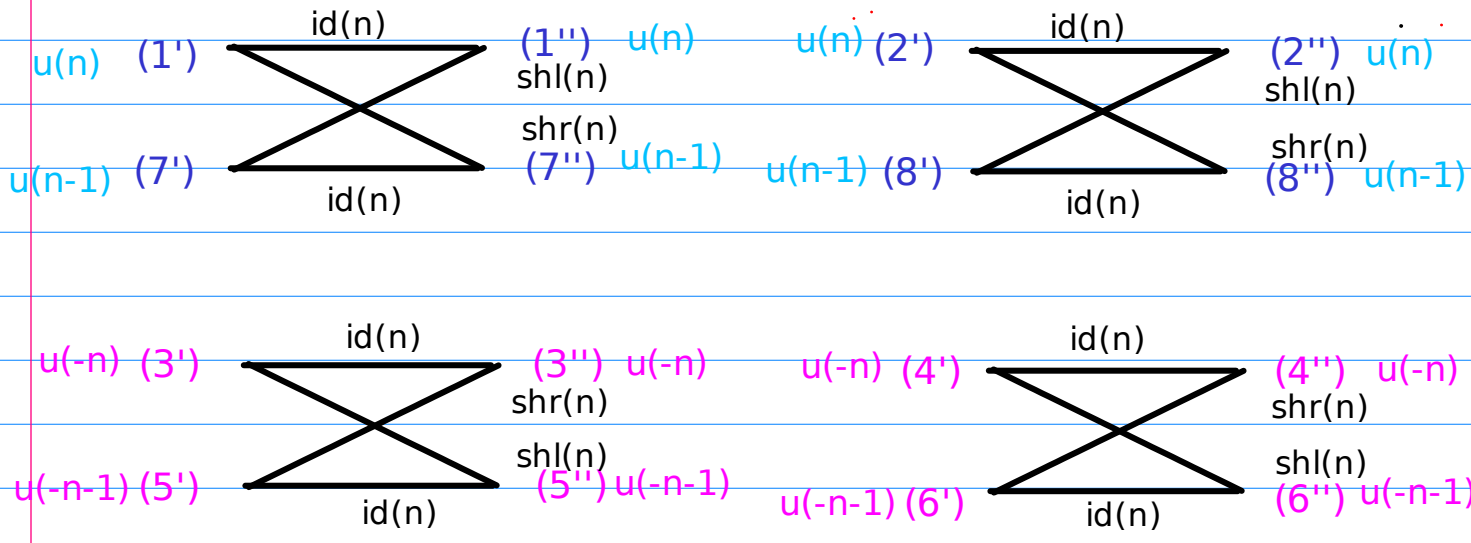
C.3 (x') → (x'')

(1')~(8') (1'')~(8'')

Exponent Conversion



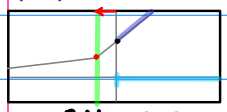
Range Conversion



C.4 (x') → (x'')

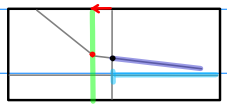
(1')~(8') (1'')~(8'')

(1')



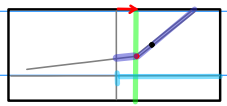
a^{n+1} u(n)

(2')



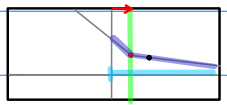
a^{-n-1} u(n)

(1'')



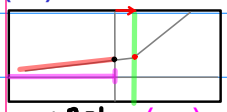
a^{n-1} u(n)

(2'')



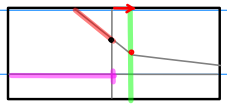
a^{-n+1} u(n)

(3')



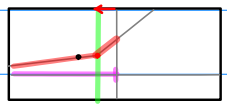
a^{n-1} u(-n)

(4')



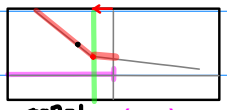
a^{-n+1} u(-n)

(3'')



a^{n+1} u(-n)

(4'')

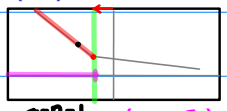


a^{-n-1} u(-n)

(5')

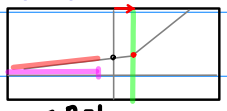
a^{n+1} u(-n-1)

(6')



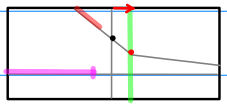
a^{-n-1} u(-n-1)

(5'')



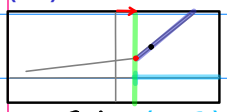
a^{n-1} u(-n-1)

(6'')



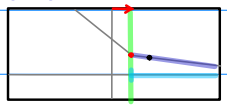
a^{-n+1} u(-n-1)

(7')



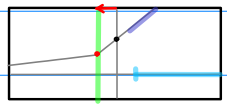
a^{n-1} u(n-1)

(8')



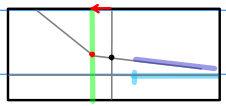
a^{-n+1} u(n-1)

(7'')



a^{n+1} u(n-1)

(8'')



a^{-n-1} u(n-1)

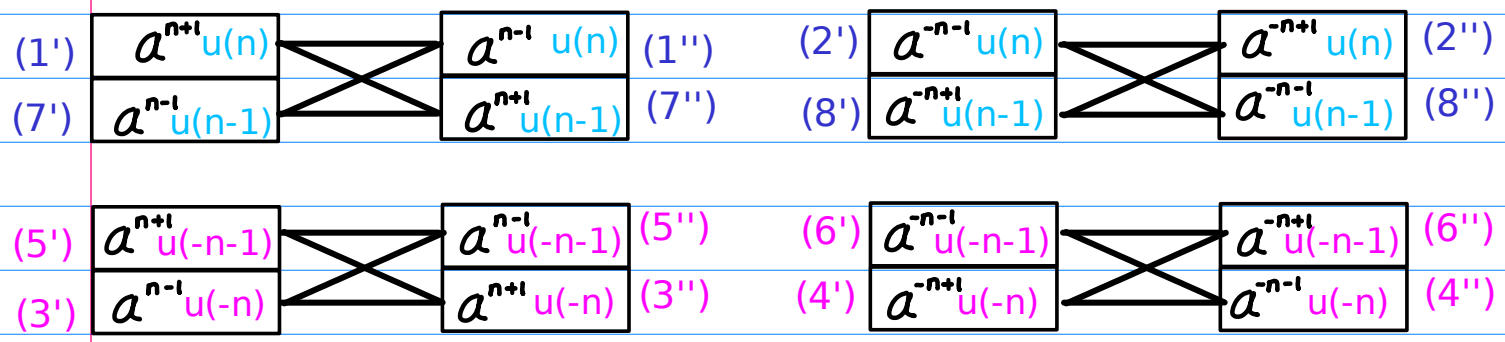
Conversion between x' and x''

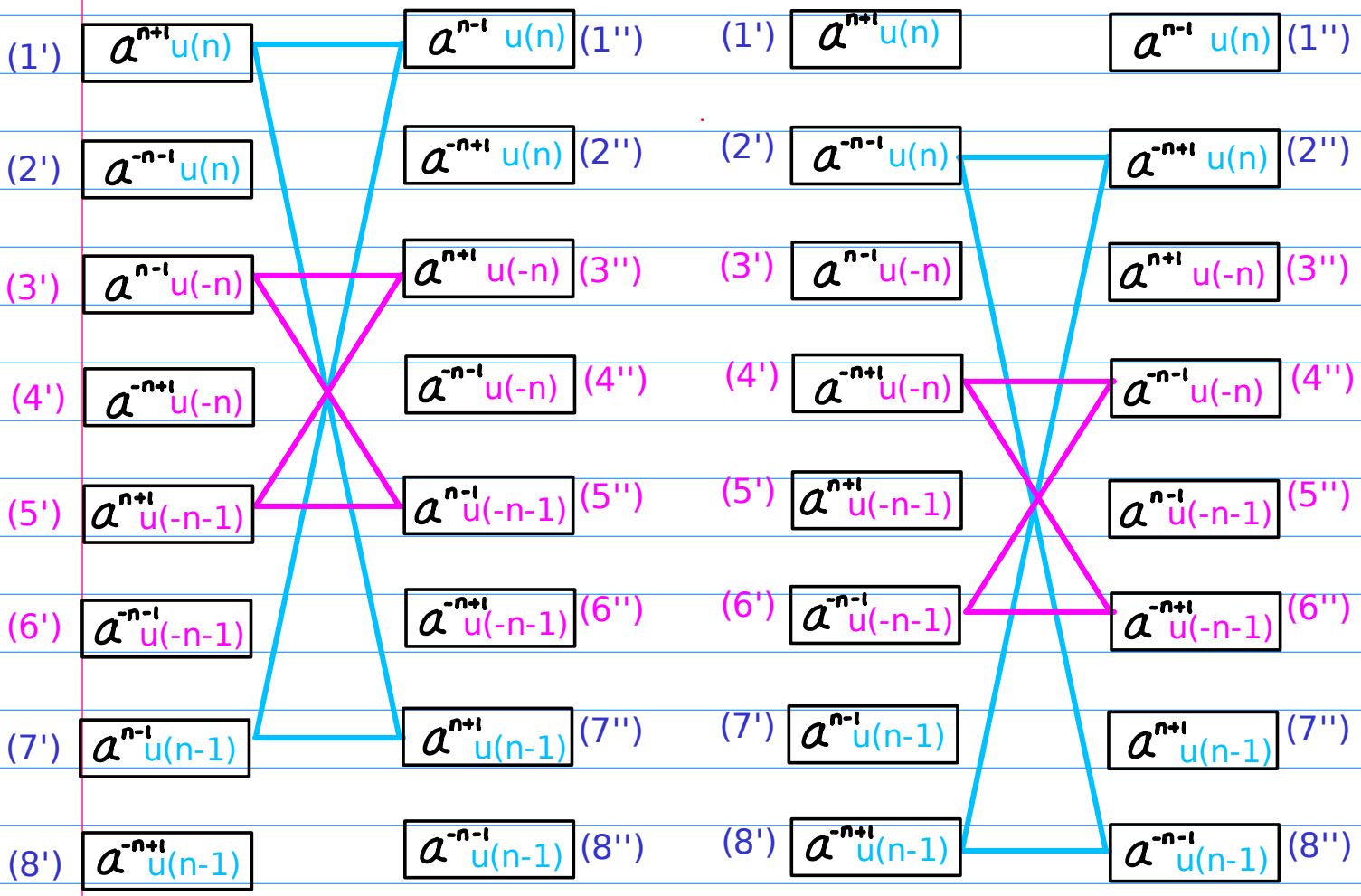
$$shl(b^n) \Longleftrightarrow shr(b^n)$$

$$shr(b^n) \Longleftrightarrow shl(b^n)$$

C.5 (x') → (x'')

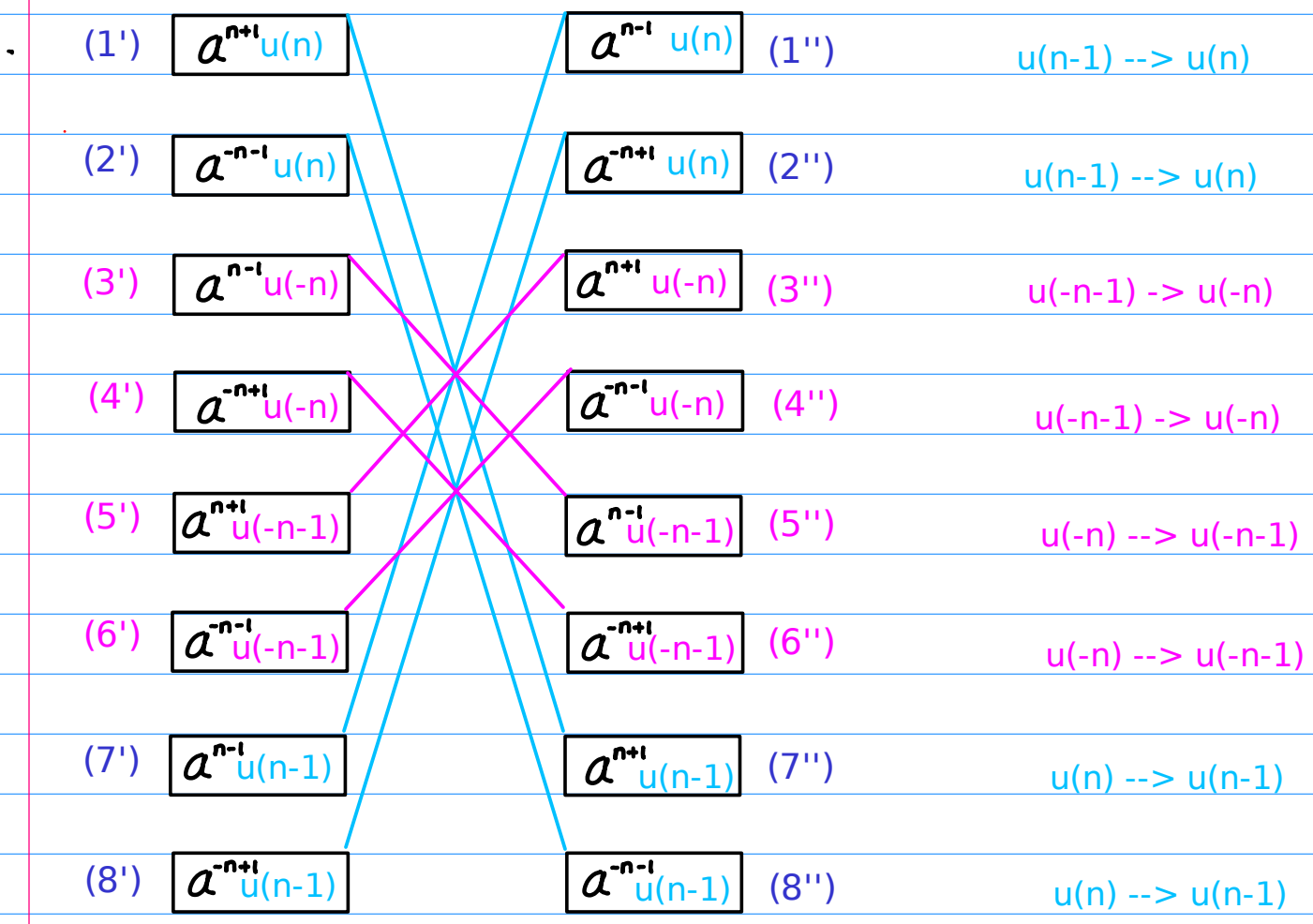
(1')~(8') (1'')~(8'')


(1') $a^{n+l}u(n)$
(7') $a^{n-l}u(n-1)$
$a^{n-l}u(n)$ (1'')
$a^{n+l}u(n-1)$ (7'')
(5') $a^{n+l}u(-n-1)$
(3') $a^{n-l}u(-n)$
$a^{n-l}u(-n-1)$ (5'')
$a^{n+l}u(-n)$ (3'')
(2') $a^{-n-l}u(n)$
(8') $a^{-n+l}u(n-1)$
$a^{-n+l}u(n)$ (2'')
$a^{-n-l}u(n-1)$ (8'')
(6') $a^{-n-l}u(-n-1)$
(4') $a^{-n+l}u(-n)$
$a^{-n+l}u(-n-1)$ (6'')
$a^{-n-l}u(-n)$ (4'')

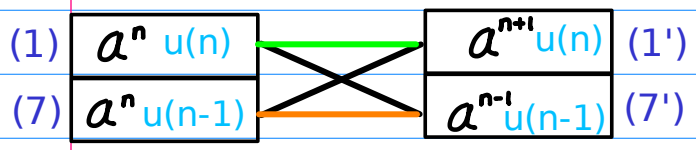

(1') $a^{n+l}u(n)$
(2') $a^{-n-l}u(n)$
(3') $a^{n-l}u(-n)$
(4') $a^{-n+l}u(-n)$
(5') $a^{n+l}u(-n-1)$
(6') $a^{-n-l}u(-n-1)$
(7') $a^{n-l}u(n-1)$
(8') $a^{-n+l}u(n-1)$
$a^{n-l}u(n)$ (1'')
$a^{-n+l}u(n)$ (2'')
$a^{n+l}u(-n)$ (3'')
$a^{-n-l}u(-n)$ (4'')
$a^{n-l}u(-n-1)$ (5'')
$a^{-n+l}u(-n-1)$ (6'')
$a^{n+l}u(n-1)$ (7'')
$a^{-n-l}u(n-1)$ (8'')

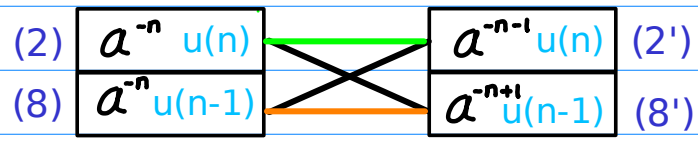
Shifted Sequence 1 x'

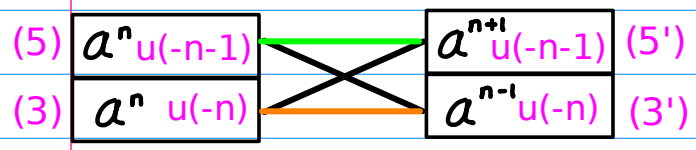
Shifted Sequence 2 x''



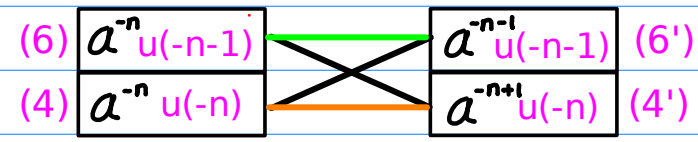
A Shifting Shifted Sequence 1





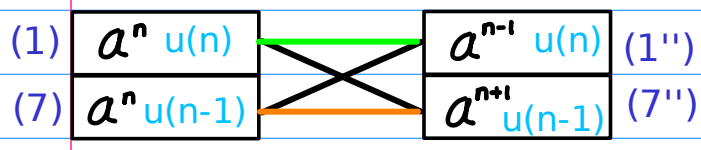


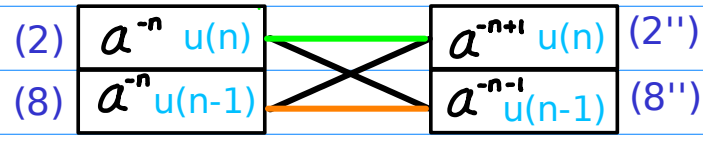
$b = a$

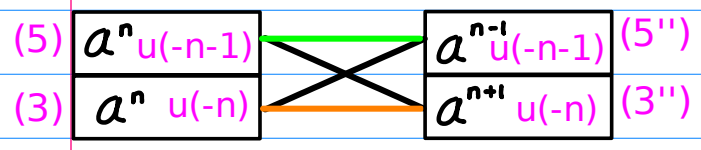


$b = a^{-1}$

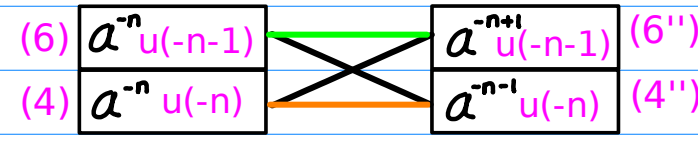
B Shifting Shifted Sequence 2





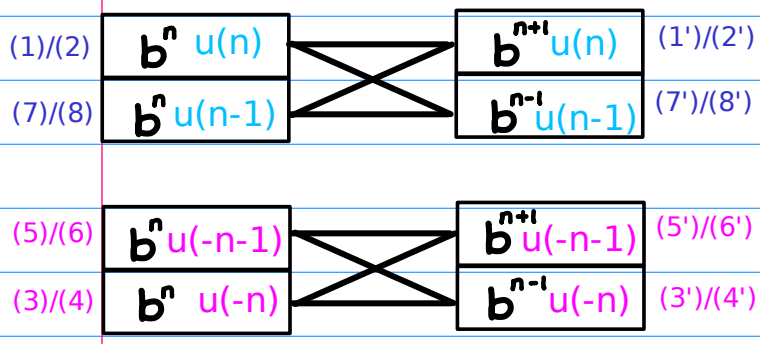


$b = a$

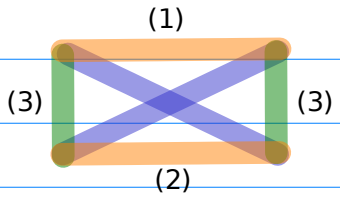


$b = a^{-1}$

A Shifting Shifted Sequence 1
(1) Exponent Shifting
(2) Range Shifting



$b = a/a^{-1}$



$$b^n \overset{(1)}{\Longleftrightarrow} b^{n+1}$$
$$b^n \overset{(2)}{\Longleftrightarrow} b^{n-1}$$
$$R(n) \overset{(3)}{\Longleftrightarrow} R(n-1)$$

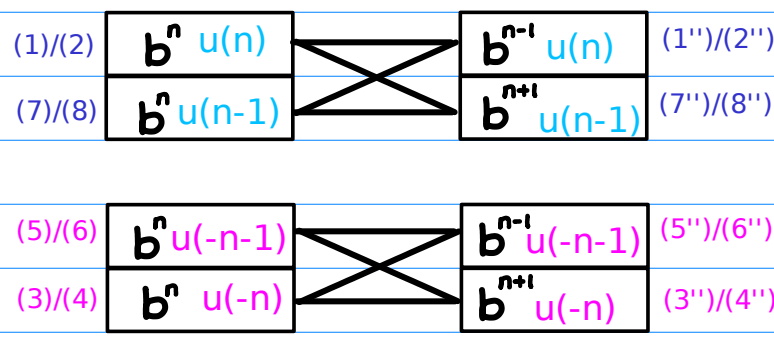
Exponent Shifting

b^n	$b^{sh(n)}$	
a^n	$a^{(n+1)}$	$shl(b^n)$
a^n	$a^{(n-1)}$	$shr(b^n)$
a^{-n}	$a^{-(n+1)}$	$shl(b^n)$
a^{-n}	$a^{-(n-1)}$	$shr(b^n)$

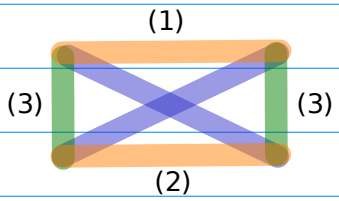
Range Shifting

$R(n)$	$R(sh(n))$	
$u(n)$	$u(n-1)$	$shr(R(n))$
$u(-(n+1))$	$u(-n)$	$shr(R(n))$

B Shifting Shifted Sequence 2
(1) Exponent Shifting
(2) Range Shifting



$b = a/a^{-1}$



$$b^n \overset{(1)}{\Longleftrightarrow} b^{n-1}$$
$$b^n \overset{(2)}{\Longleftrightarrow} b^{n+1}$$
$$R(n) \overset{(3)}{\Longleftrightarrow} R(n-1)$$

Exponent Shifting

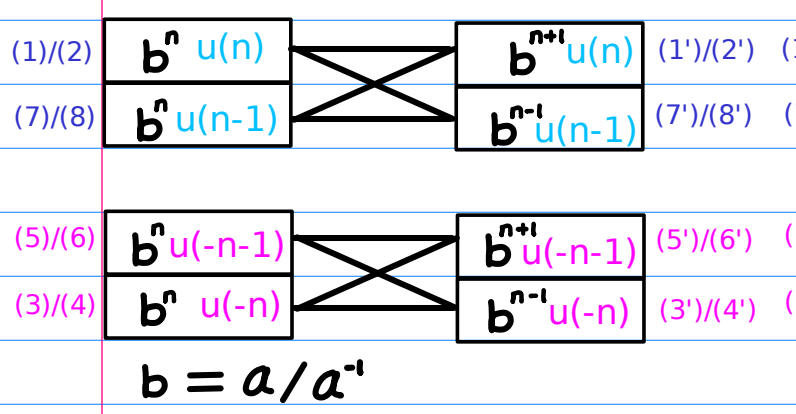
b^n	$b^{sh(n)}$	
a^n	$a^{(n-1)}$	$shr(b^n)$
a^n	$a^{(n+1)}$	$shl(b^n)$
a^{-n}	$a^{-(n-1)}$	$shr(b^n)$
a^{-n}	$a^{-(n+1)}$	$shl(b^n)$

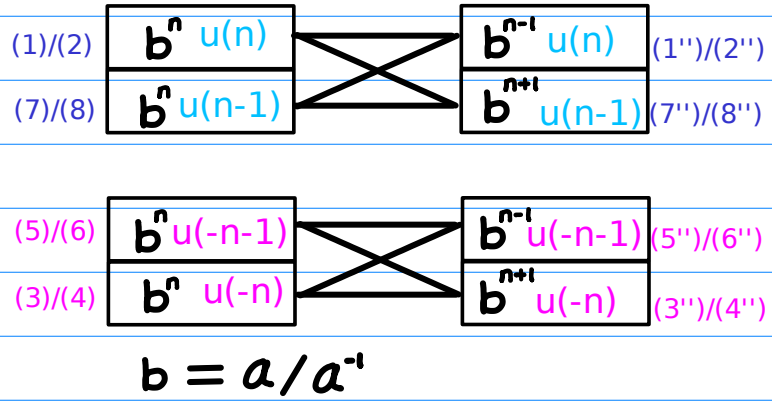
Range Shifting

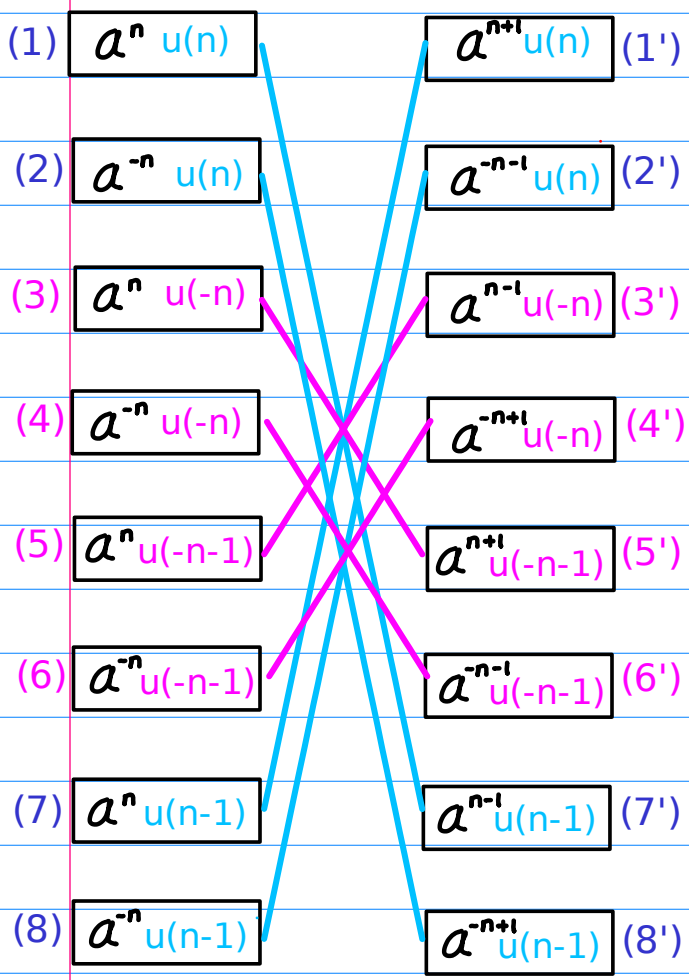
$R(n)$	$R(sh(n))$	
$u(n)$	$u(n-1)$	$shr(R(n))$
$u(-(n+1))$	$u(-n)$	$shr(R(n))$

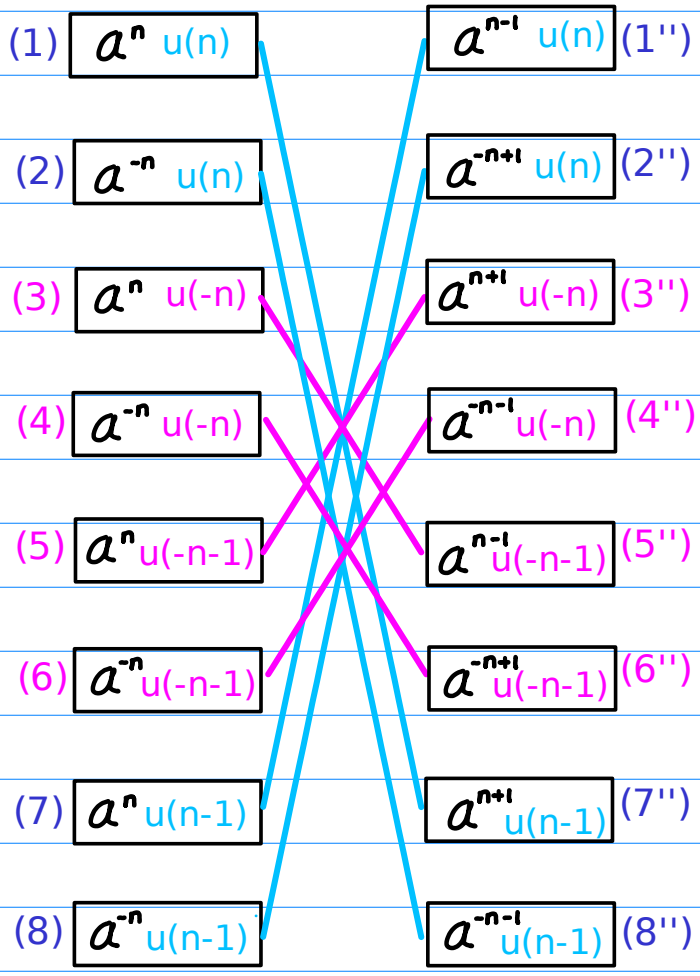
A Shifting **Shifted Sequence 1**

B Shifting **Shifted Sequence 2**



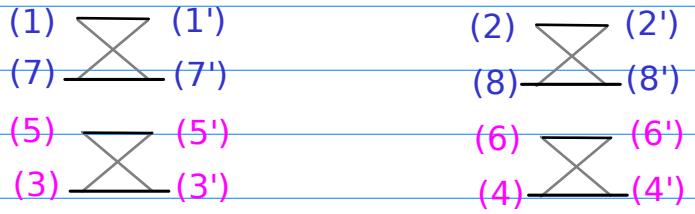


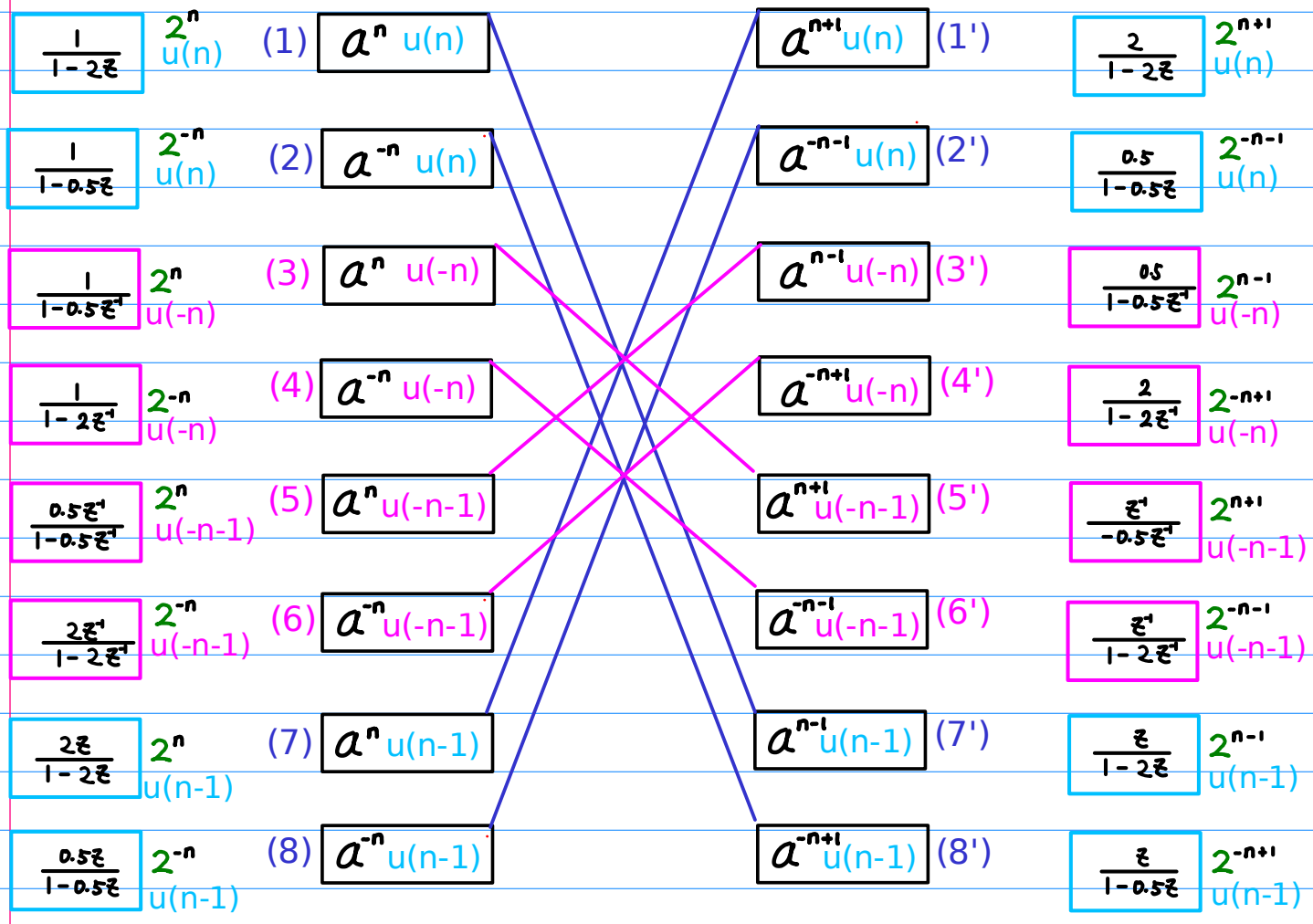




A Shifting to Shifted Sequence 1

Unshifted Sequence x → **Shifted Sequence 1 x'**



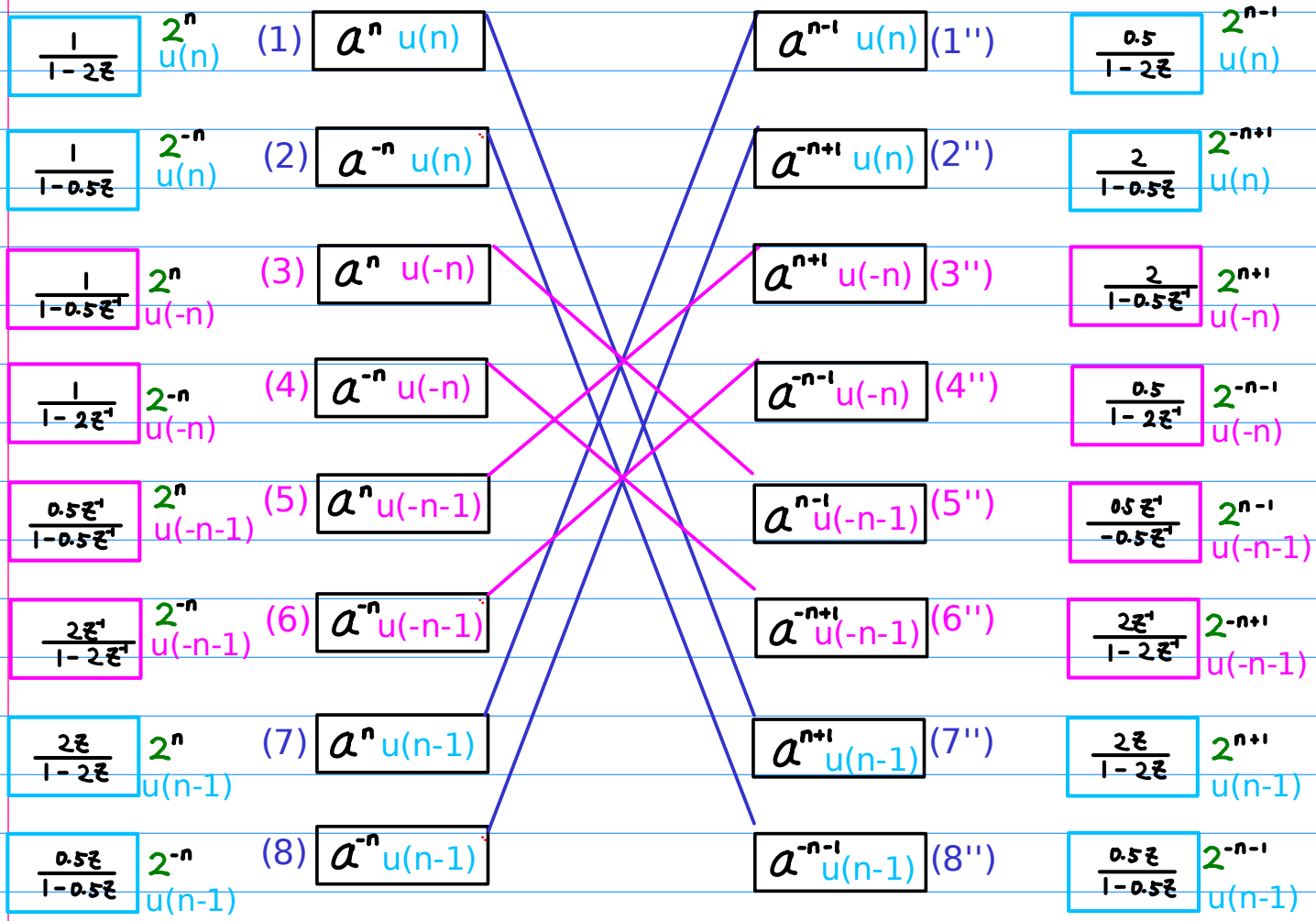


B Shifting to Shifted Sequence 2

Unshifted Sequence x → **Shifted Sequence 2 x''**

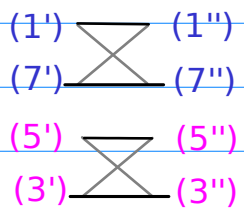
(1) ⟷ (1'')
(7) ⟷ (7'')
(5) ⟷ (5'')
(3) ⟷ (3'')

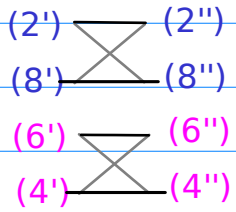
(2) ⟷ (2'')
(8) ⟷ (8'')
(6) ⟷ (6'')
(4) ⟷ (4'')

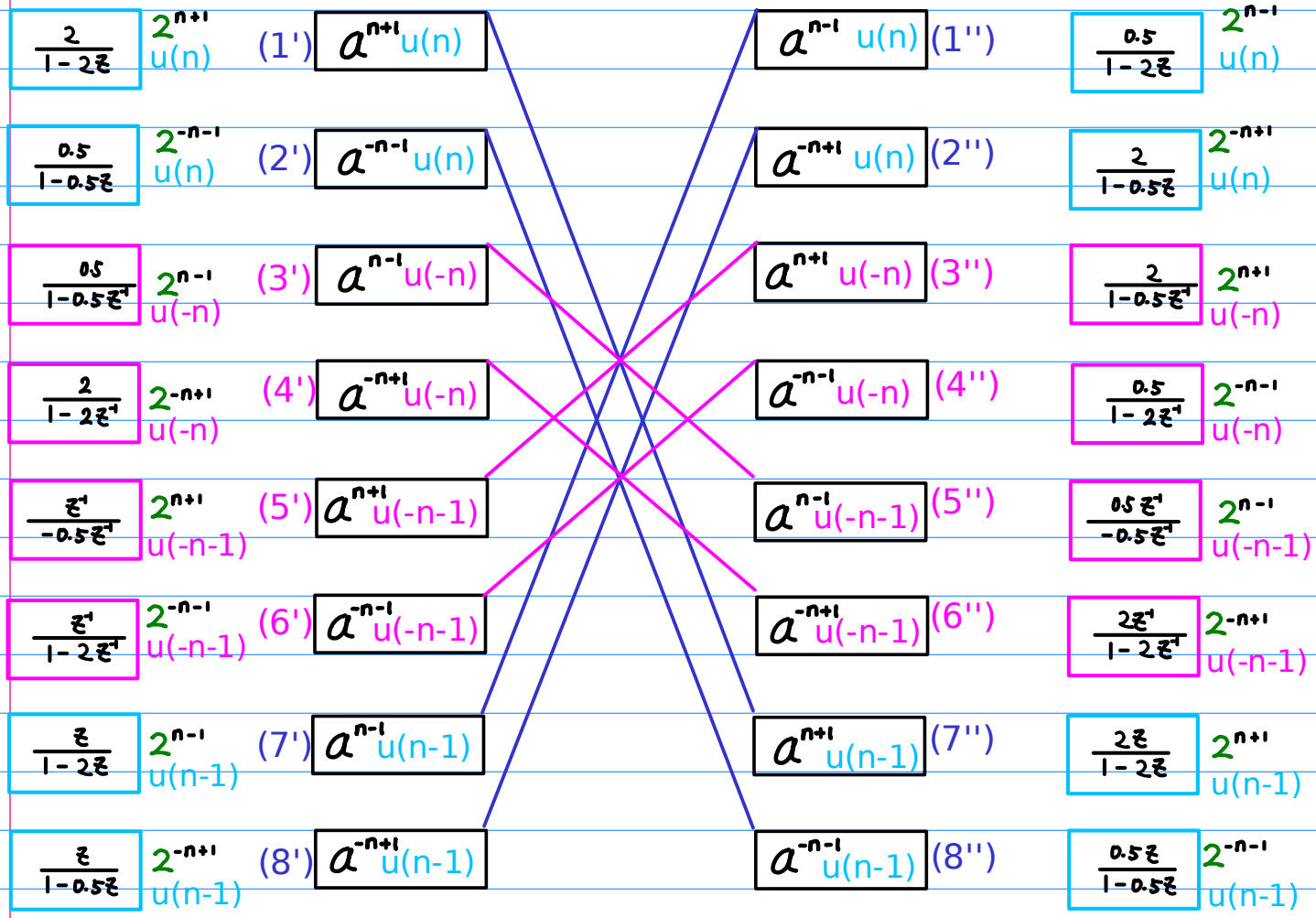


C Shifting from Shifted Sequence 1 to Shifted Sequence 2

Shifted Seq1 x' → **Shifted Seq 2 x''**

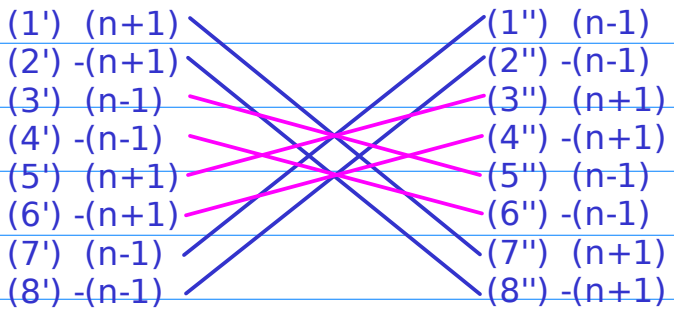






$shl(b^n) \Longleftrightarrow shr(b^n)$

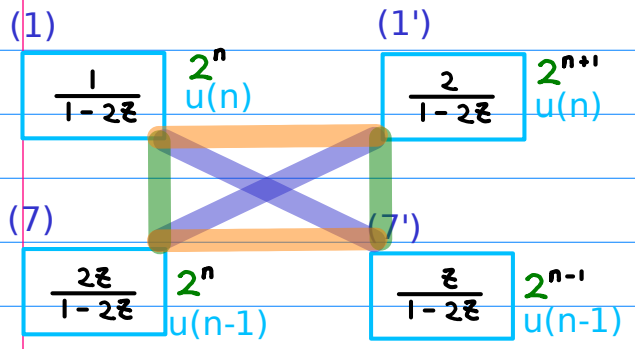
$shr(b^n) \Longleftrightarrow shl(b^n)$

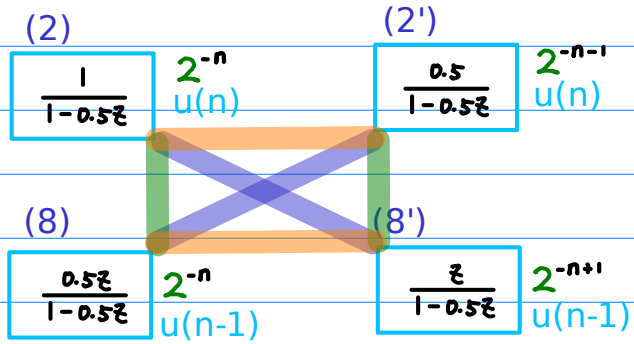


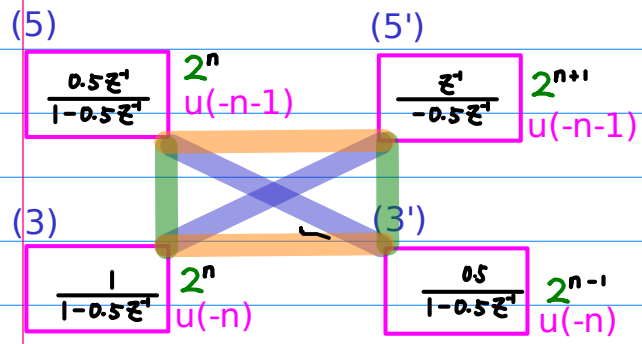
Inter-permutation (x) → (x')

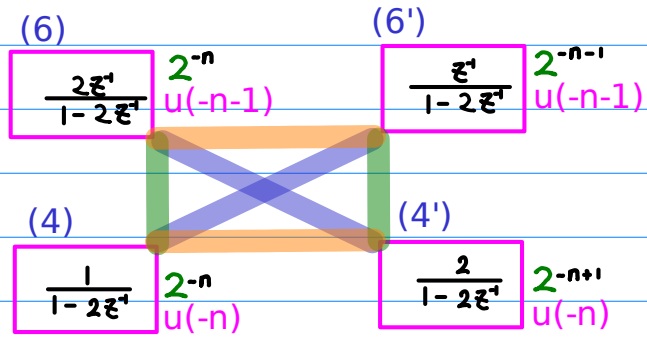
A Shifting Shifted Sequence 1

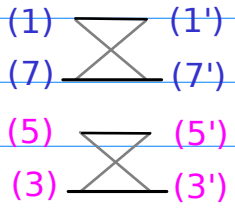
(1)~(8) (1')~(8')

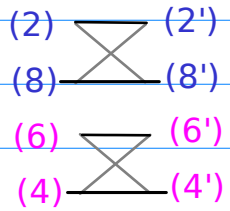


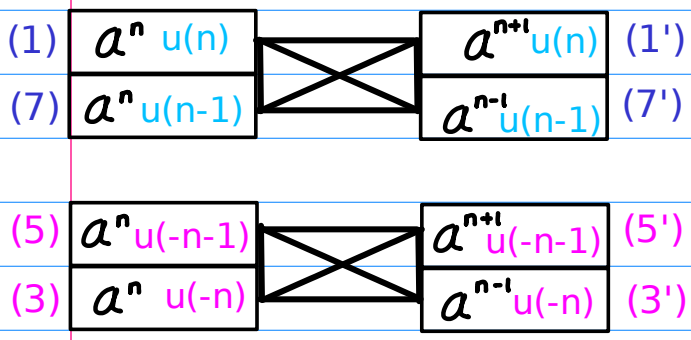


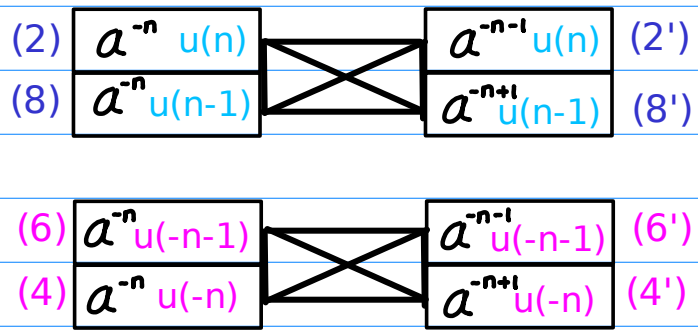








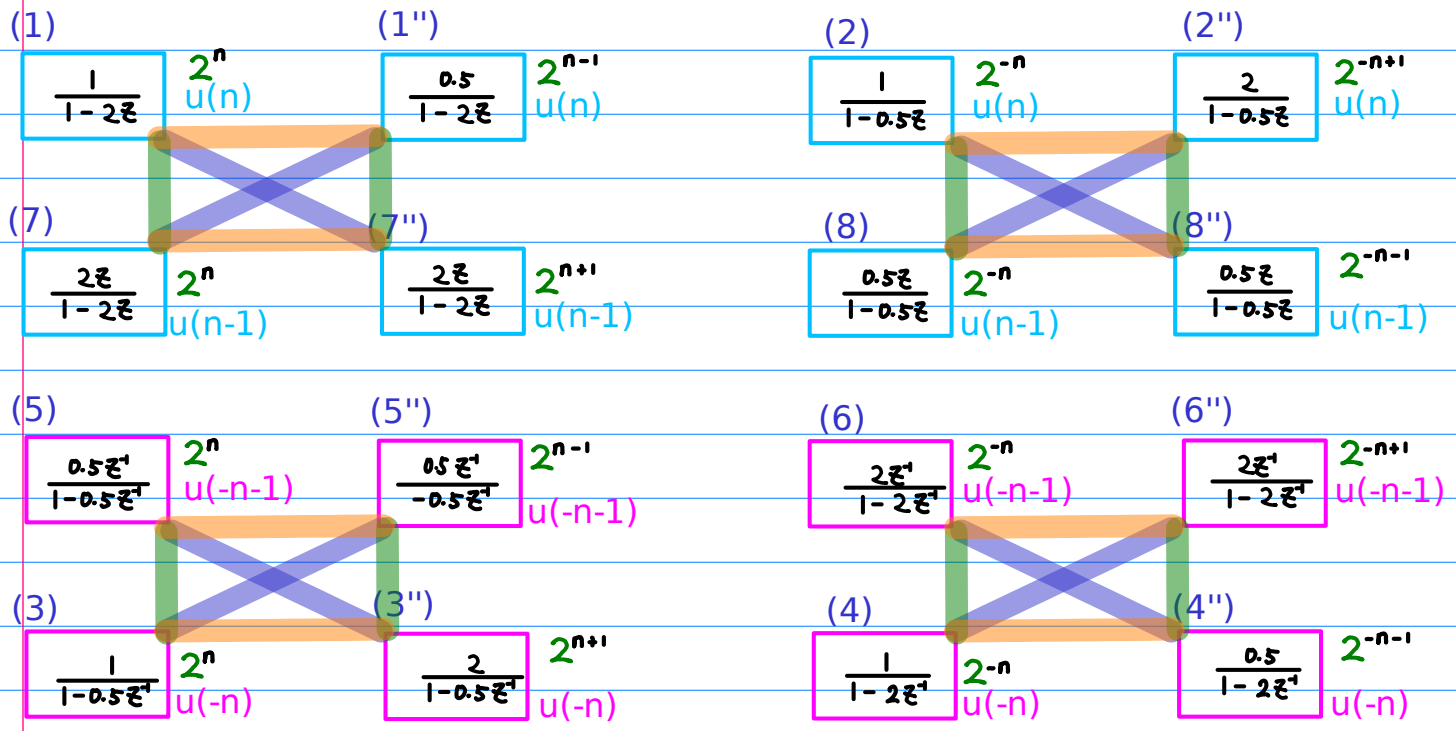




Inter-permutation (x) → (x'')

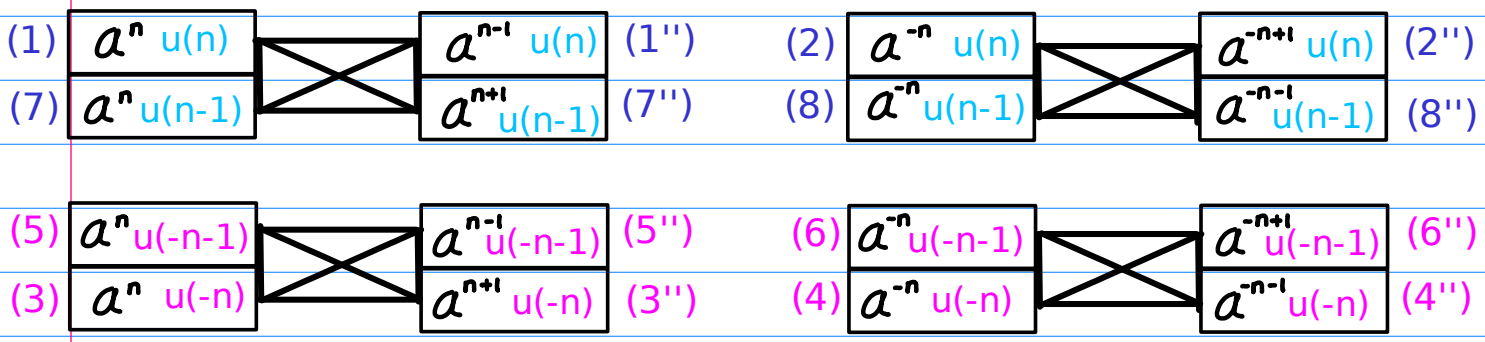
B Shifting Shifted Sequence 2

(1)~(8) → (1'')~(8'')

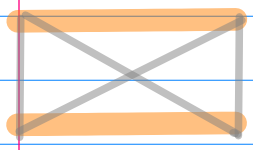


(1) (1'')
(7) (7'')
(5) (5'')
(3) (3'')

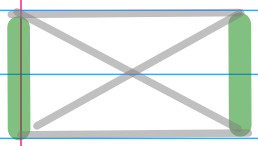
(2) (2'')
(8) (8'')
(6) (6'')
(4) (4'')



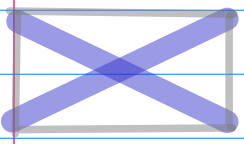
Decomposing Shift Operations



(exponent shift, identity)



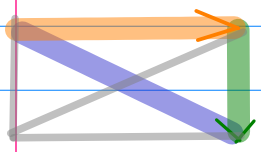
(identity, range shift)

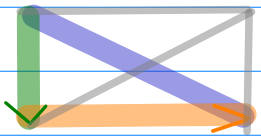


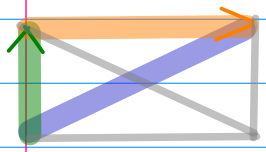
(exponent shift, range sift)

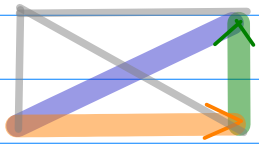
= (exponent shift, identity)

+ (identity, range shift)









A Shifting Shifted Sequence 1

(1) Exponent Shifting

(2) Range Shifting

(SR, id) + (id, SR) = (SR, SR)

(SL, id) + (id, SL) = (SL, SL)

B Shifting Shifted Sequence 2

(1) Exponent Shifting, Flipping

(2) Range Shifting

(SR, id) + (id, SL) = (SR, SL)

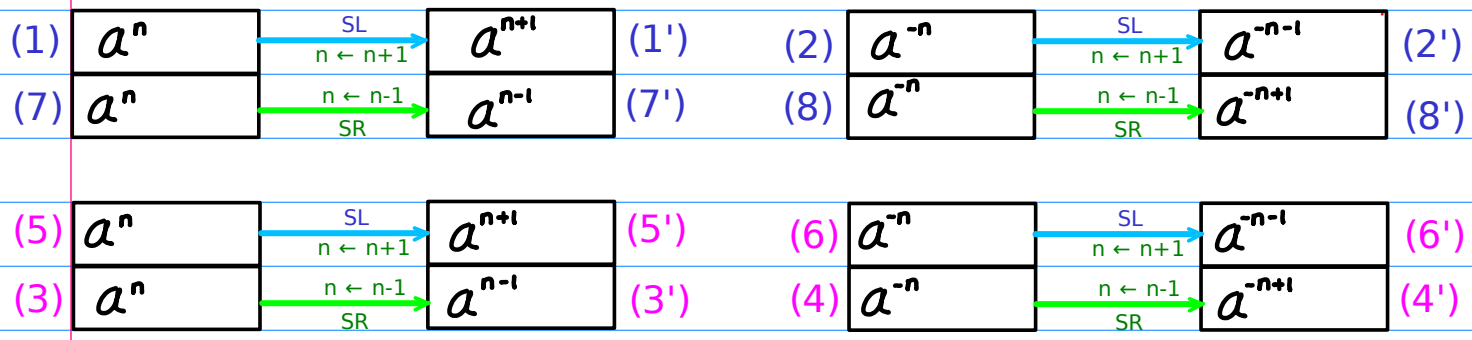
(SL, id) + (id, SR) = (SL, SR)

A Shifting **Shifted Sequence 1**

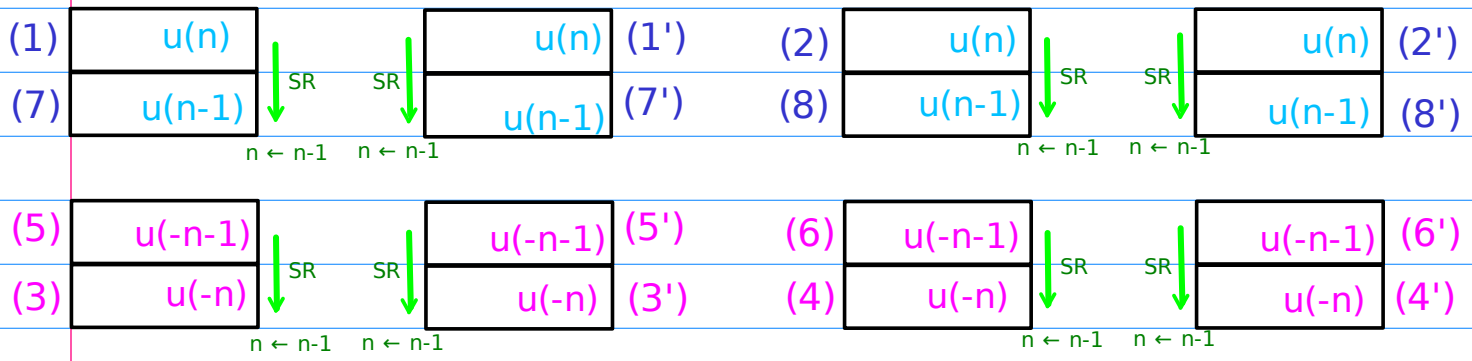
(1) Exponent Shifting **(SR, id) or (SL, id)**

(2) Range Shifting **(id, SR) or (id, SL)**

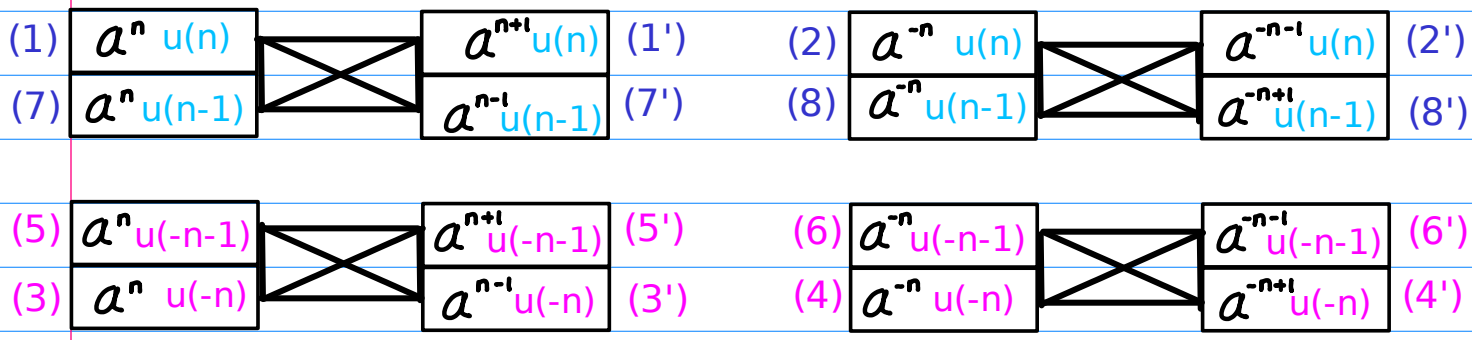
Exponent Shifts : (SR, id) or (SL, id)



Range Shifts : (id, SR) or (id, SL)



Exponent & Range Permutations



Decomposition

(EP, RP) = (EP, id) + (id, RP)

EP : Exponent Permuations

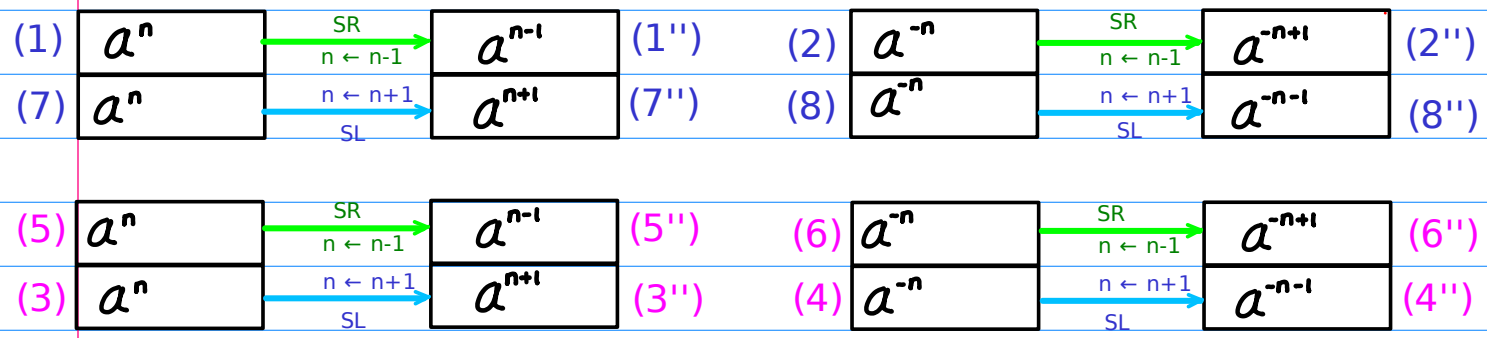
RP : Range Permutations

B Shifting Shifted Sequence 2

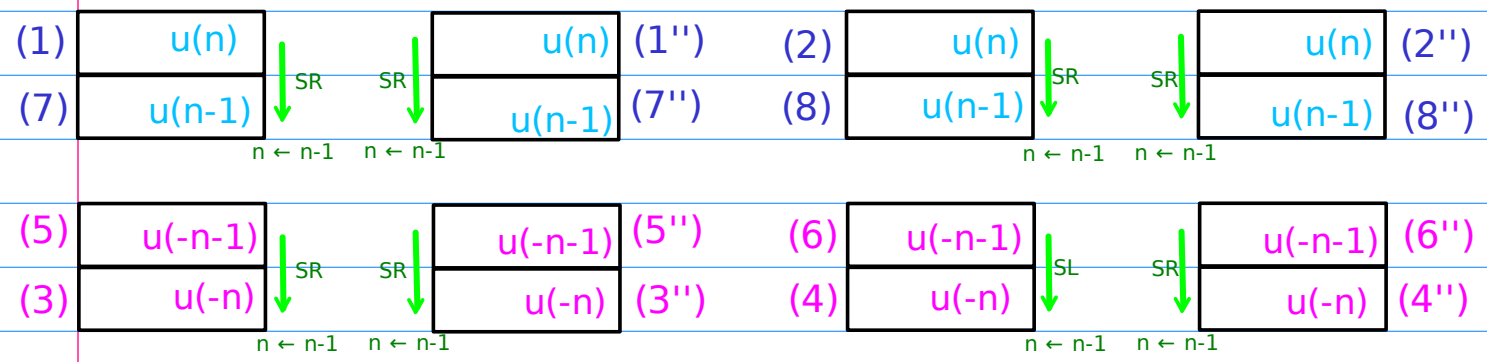
(1) Exponent Shifting, Flipping (SR, id) or (SL, id)

(2) Range Shifting (id, SR) or (id, SL)

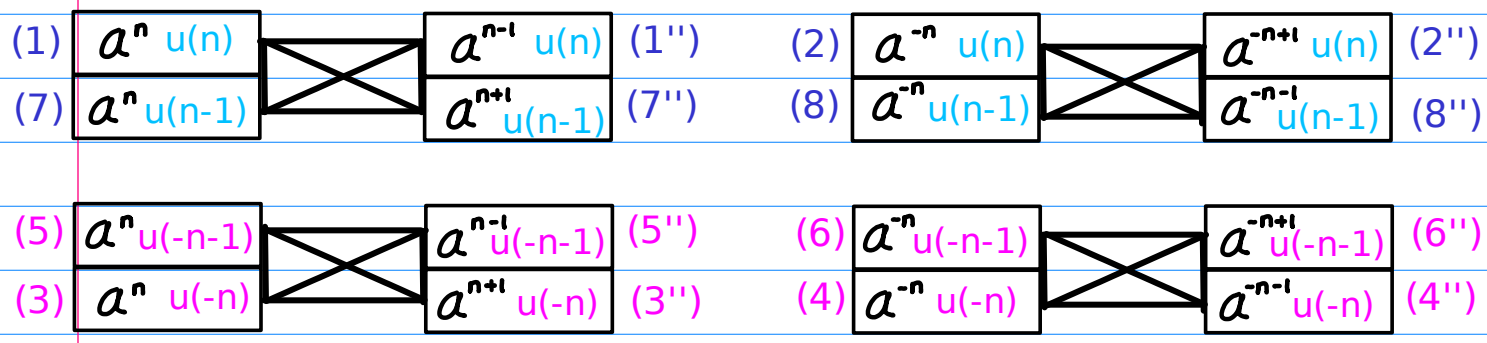
Exponent Shifts : (SR, id) or (SL, id)



Range Shifts : (id, SR) or (id, SL)



Exponent & Range Permutations



Decomposition

(EP, RP) = (EP, id) + (id, RP)

EP : Exponent Permuations

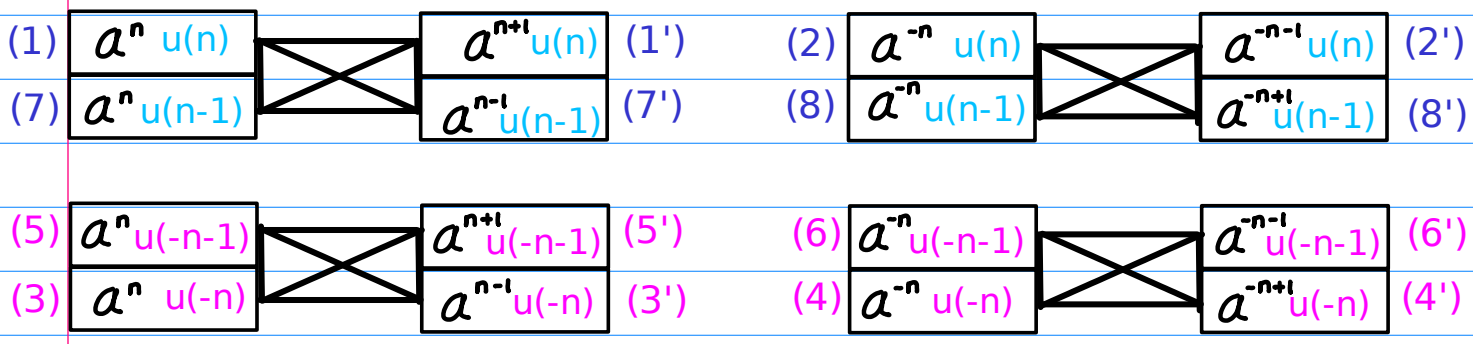
RP : Range Permutations

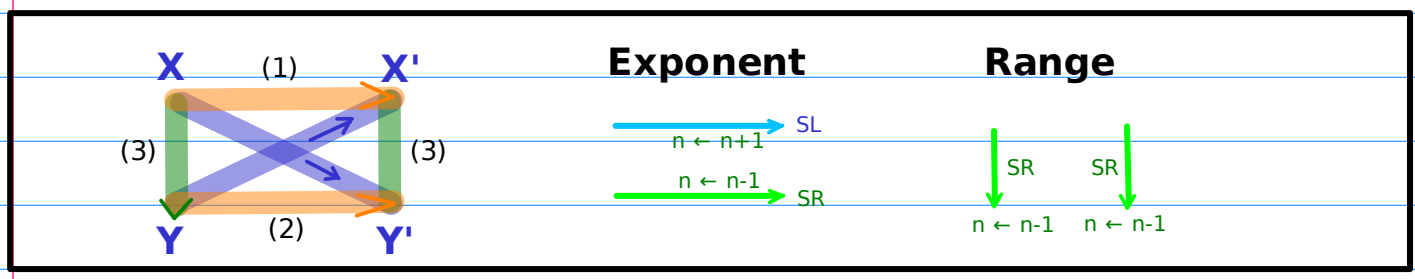
A Shifting Shifted Sequence 1

(1) Exponent Shifting
(2) Range Shifting

:

(SR, id) + (id, SR) = (SR, SR)
(SL, id) + (id, SL) = (SL, SL)

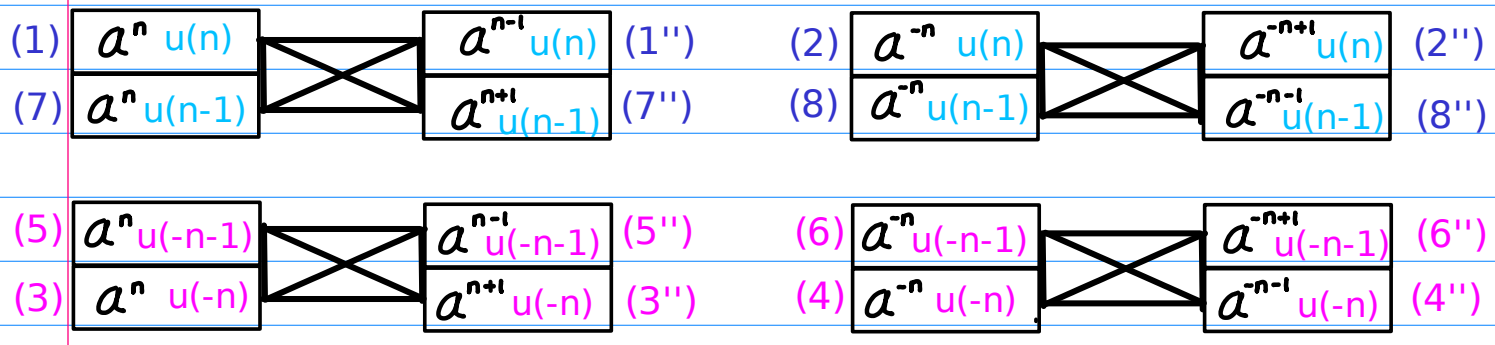


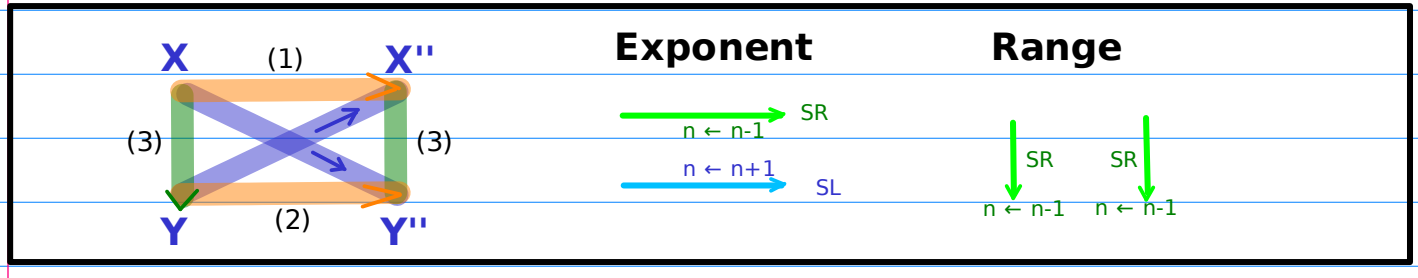


B Shifting Shifted Sequence 2

(1) Exponent Shifting
(2) Range Shifting

(SR, id) + (id, SL) = (SR, SL)
(SL, id) + (id, SR) = (SL, SR)





A Shifting **Shifted Sequence 1**

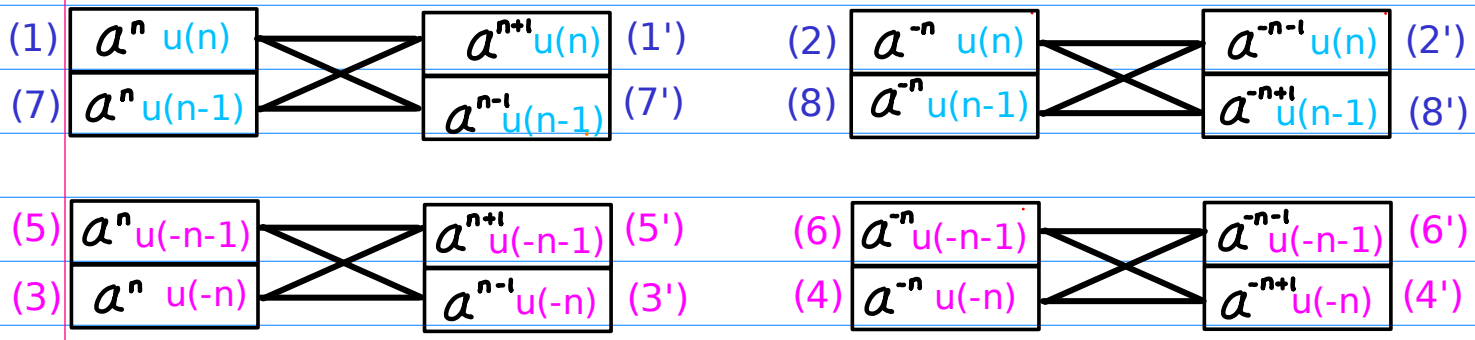
(1) Exponent Shifting

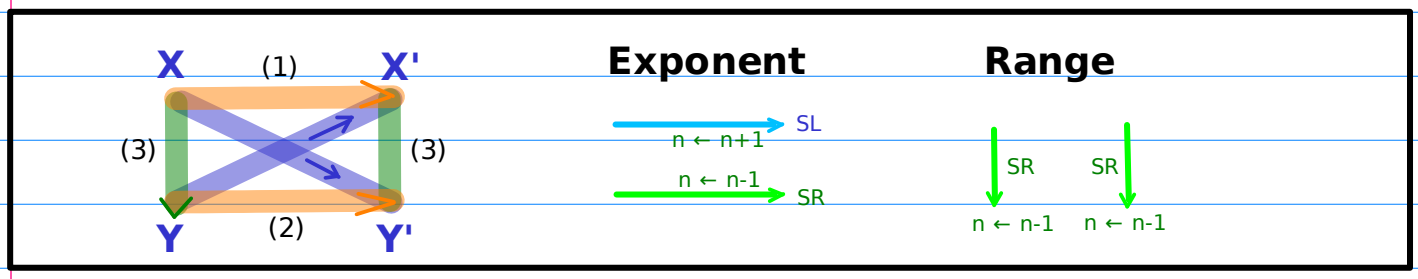
(2) Range Shifting

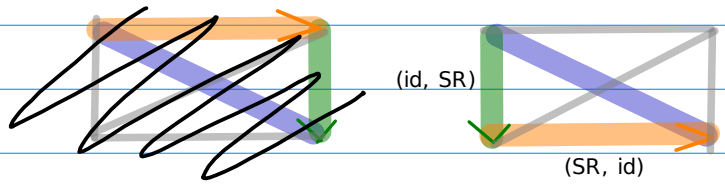
:

(SR, id) + (id, SR) = (SR, SR)

(SL, id) + (id, SL) = (SL, SL)



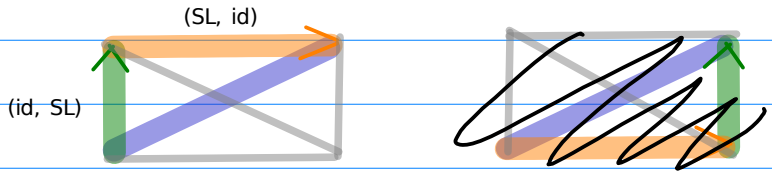




(SR, id) shift right exponent

(id, SR) shift right range

(SR, SR)



(SL, id) shift left exponent

(id, SL) shift left range

(SL, SL)

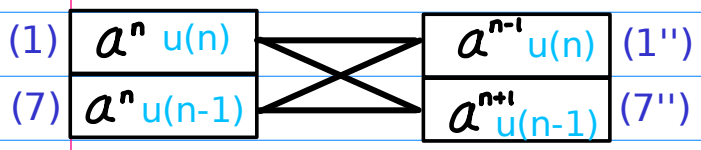
(SR, id) + (id, SR) = (SR, SR)

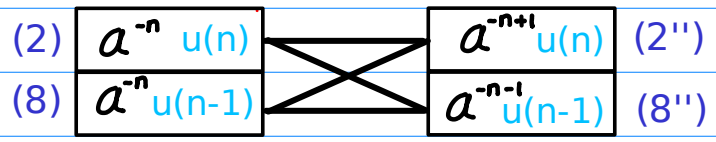
(SL, id) + (id, SL) = (SL, SL)

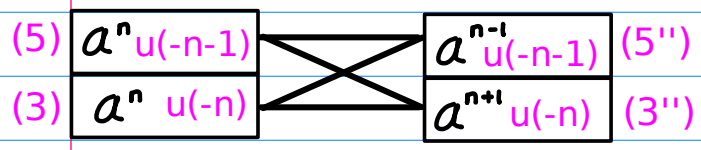
B Shifting Shifted Sequence 2

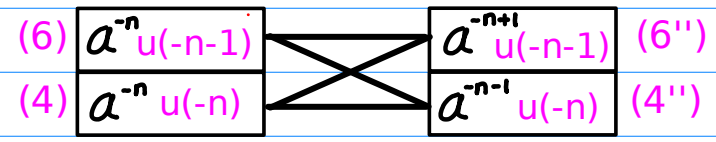
(1) Exponent Shifting
(2) Range Shifting

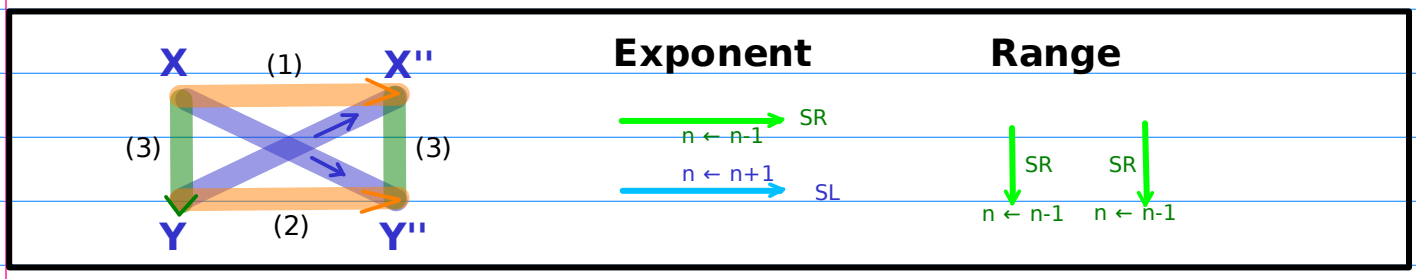
(SR, id) + (id, SL) = (SR, SL)
(SL, id) + (id, SR) = (SL, SR)

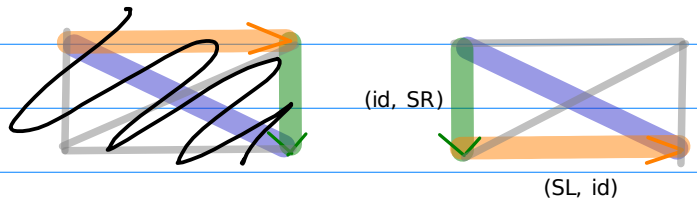








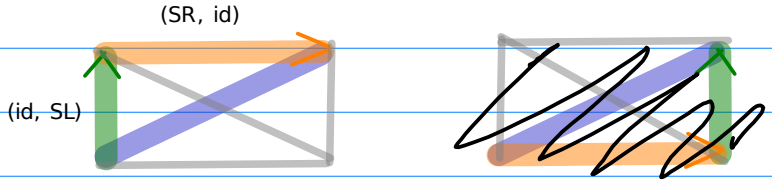




(SL, id) shift right exponent

(id, SR) shift right range

(SL, SR)



(SR, id) shift left exponent

(id, SL) shift left range

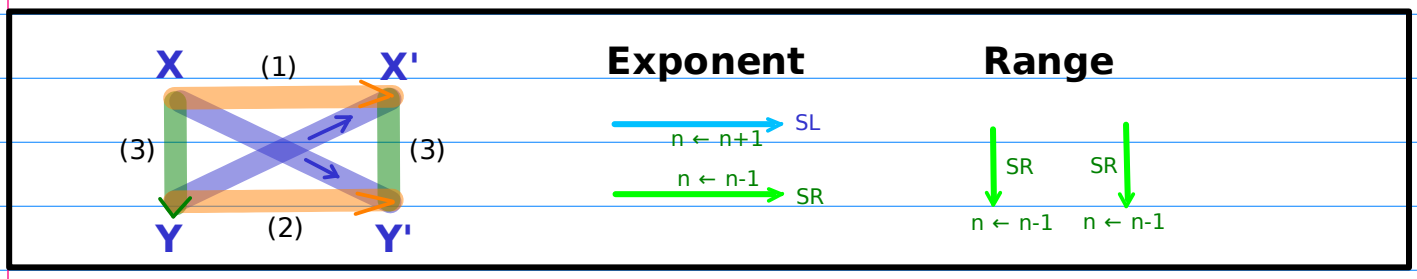
(SR, SL)

(SL, id) + (id, SR) = (SL, SR)
(SR, id) + (id, SL) = (SR, SL)

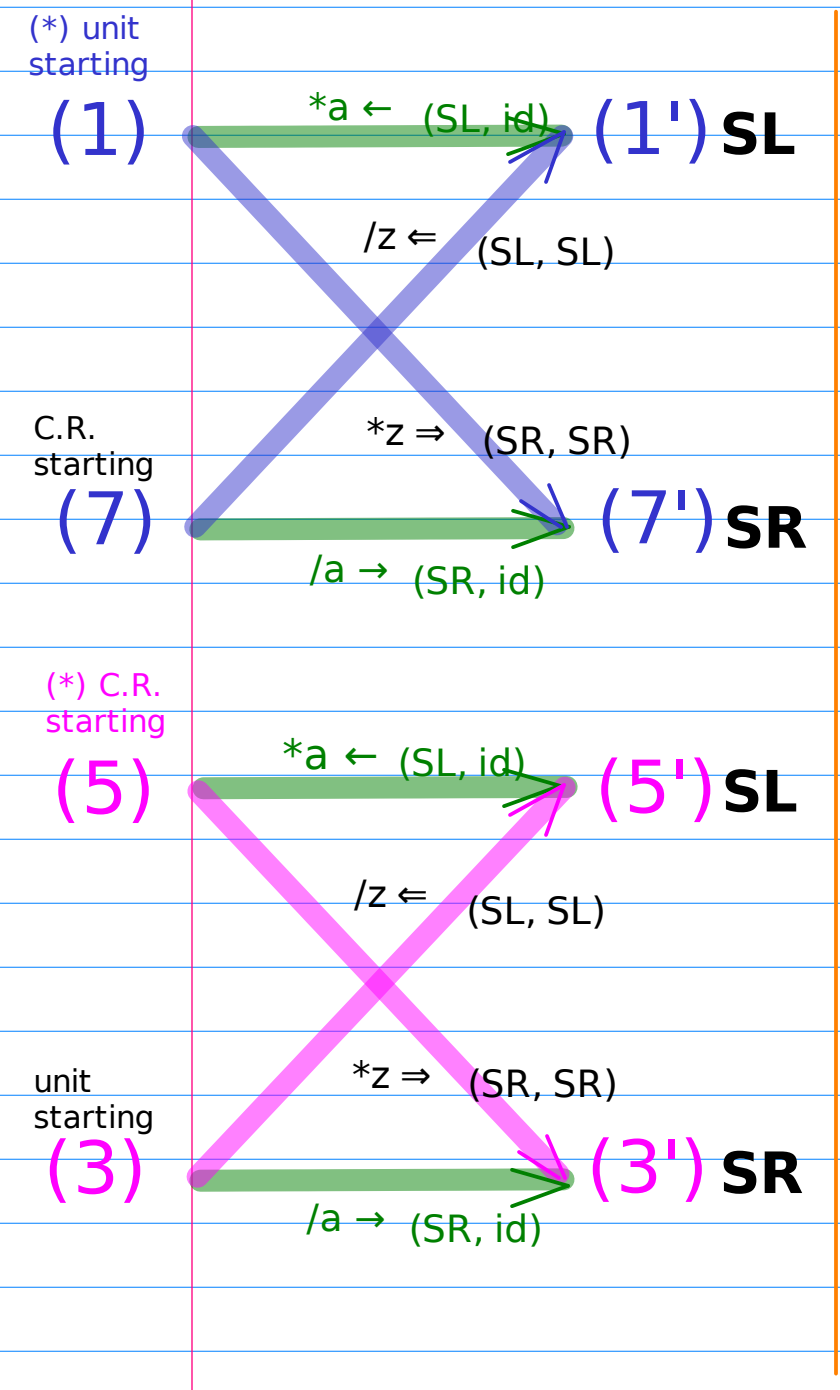
Butterfly Relations

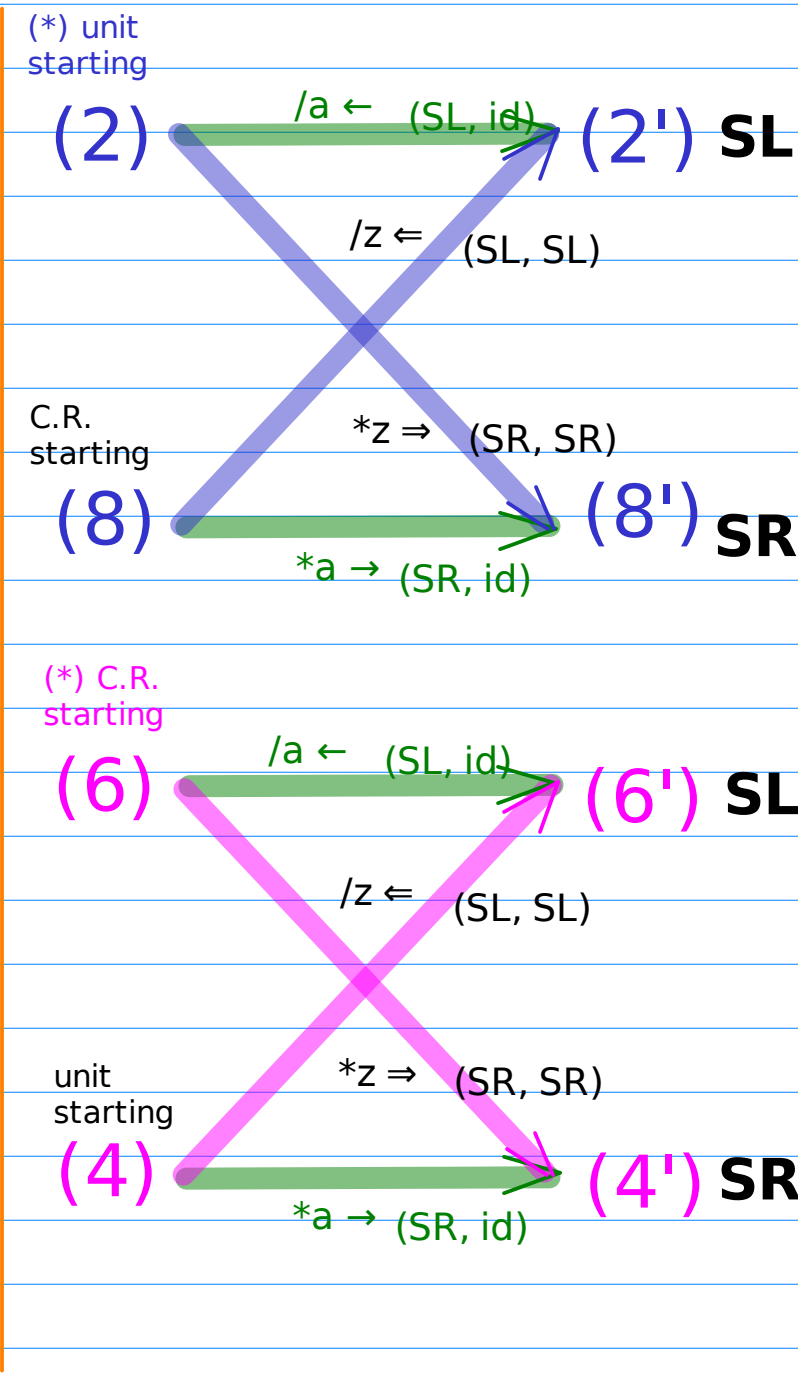
A Shifting **Shifted Sequence 1**

Unshifted Sequence x → **Shifted Sequence 1** x'



.

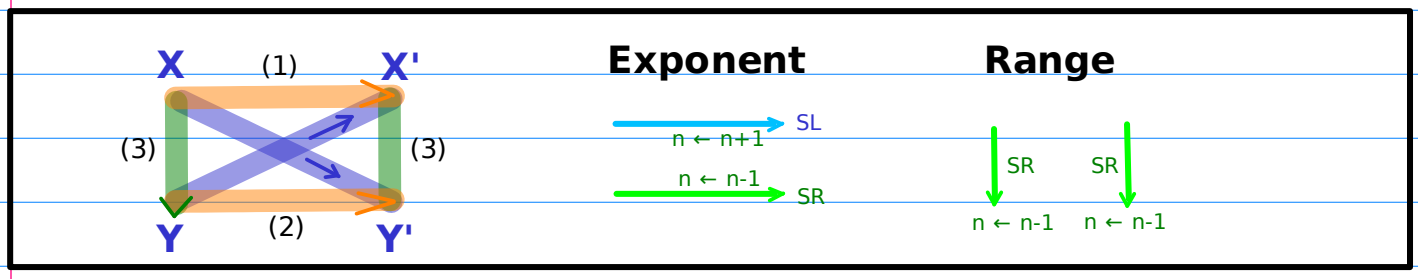


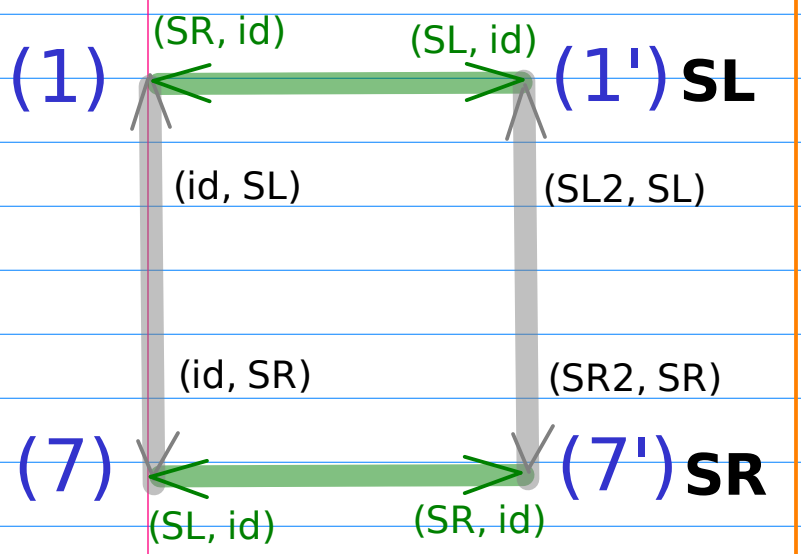


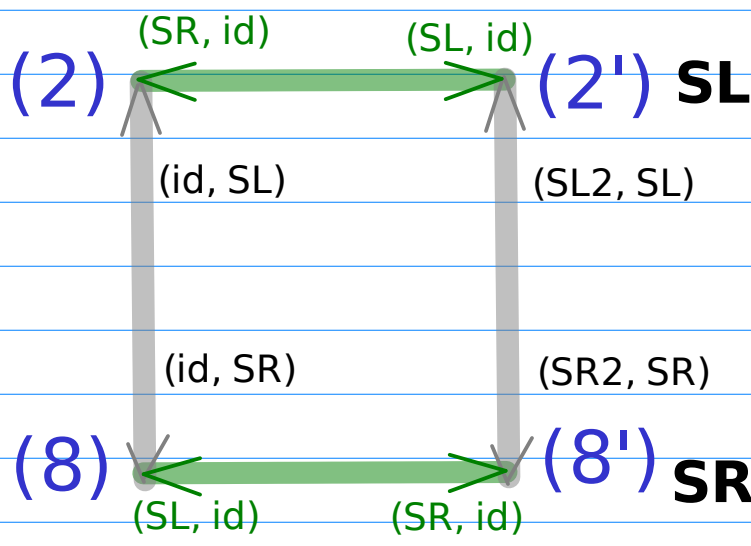
Butterfly Relations

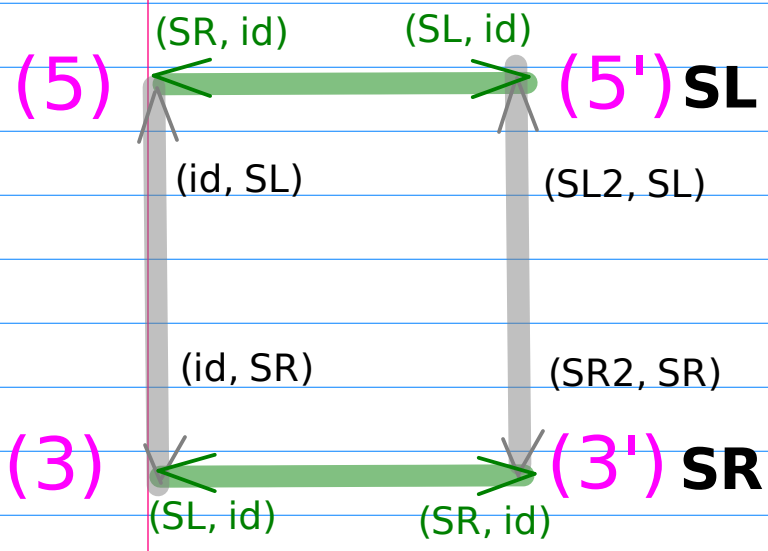
A Shifting **Shifted Sequence 1**

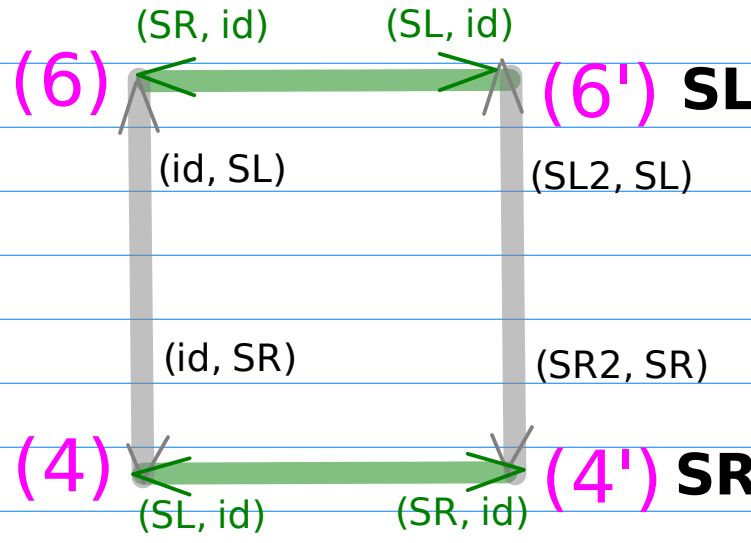
Unshifted Sequence x → **Shifted Sequence 1** x'









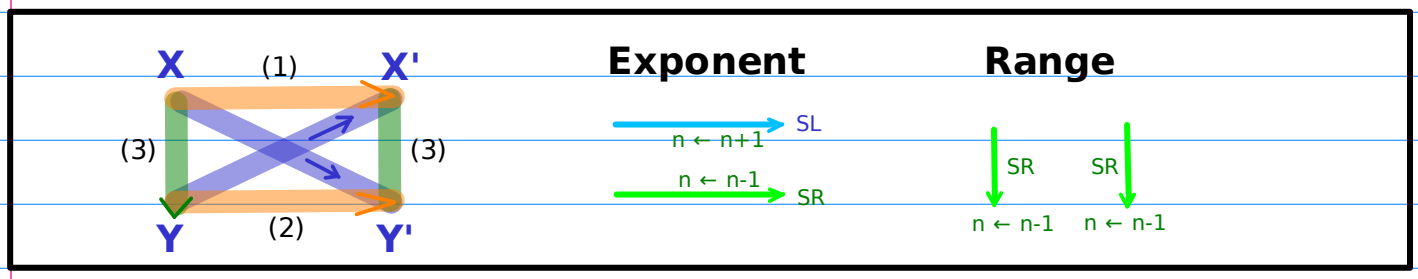


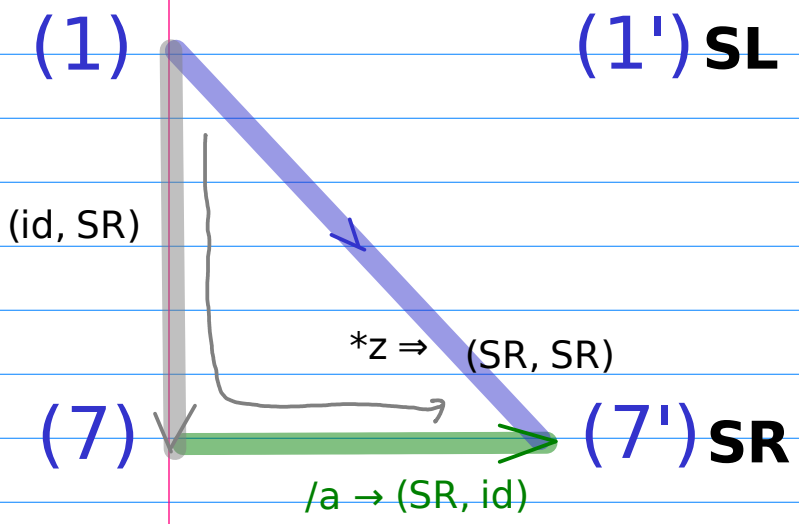
Decomposition of Exp and Rng Shifts (1)

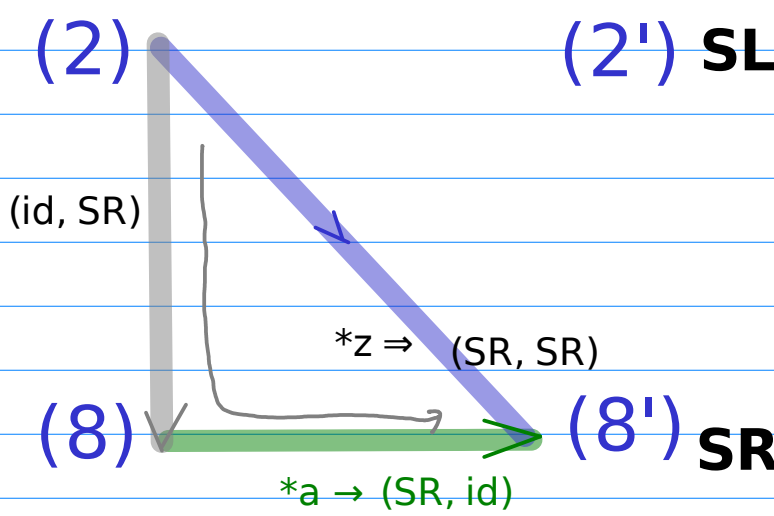
A Shifting **Shifted Sequence 1**

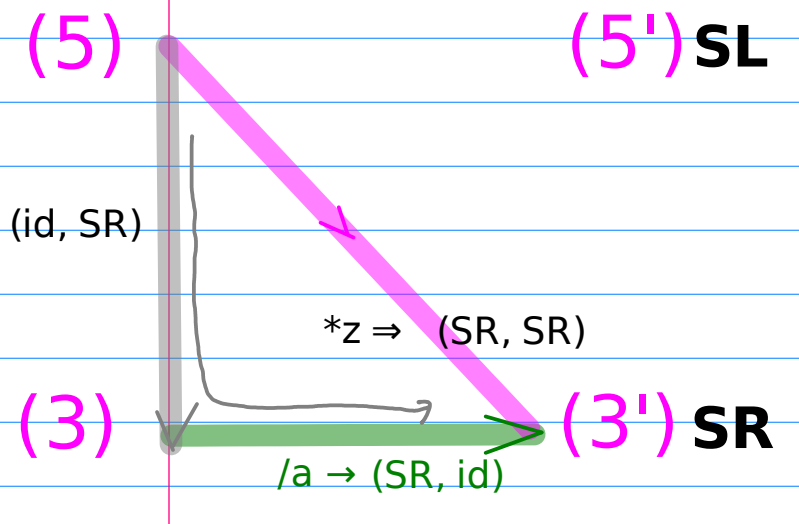
Unshifted Sequence x → **Shifted Sequence 1** x'

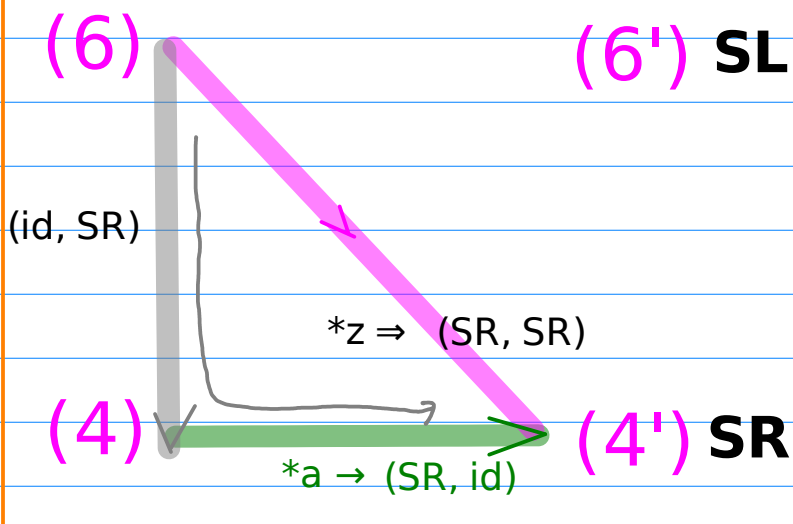
(id, SR) + (SR, id) = (SR, SR)









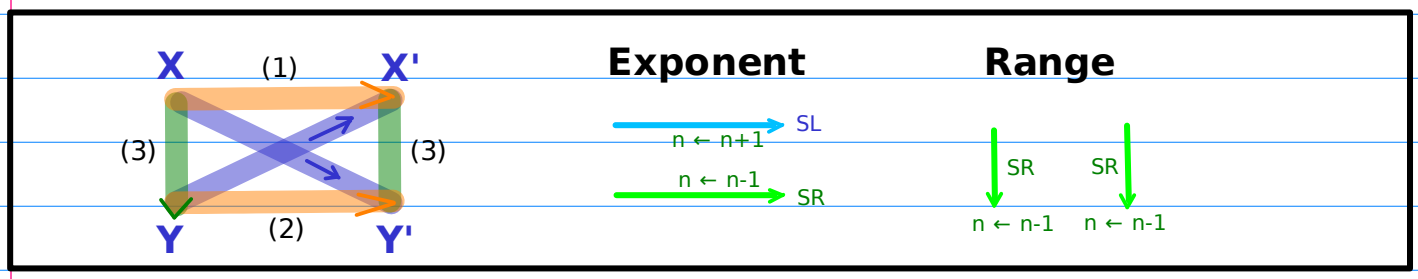


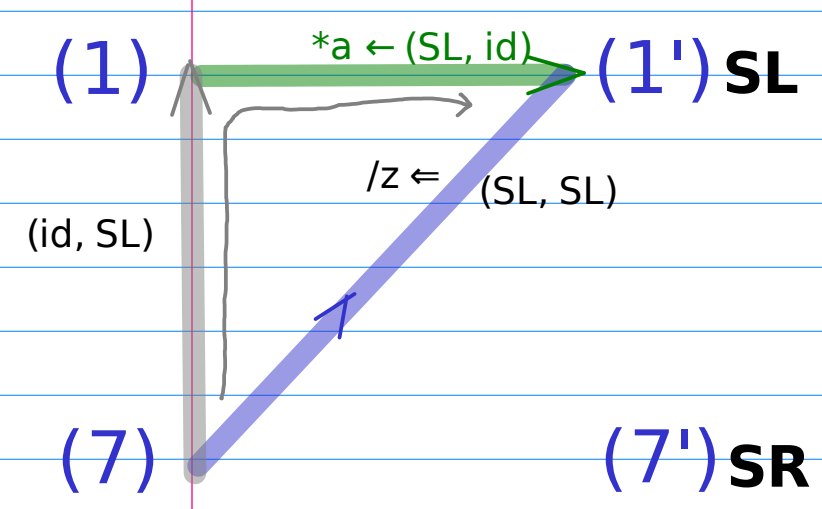
Decomposition of Exp and Rng Shifts (2)

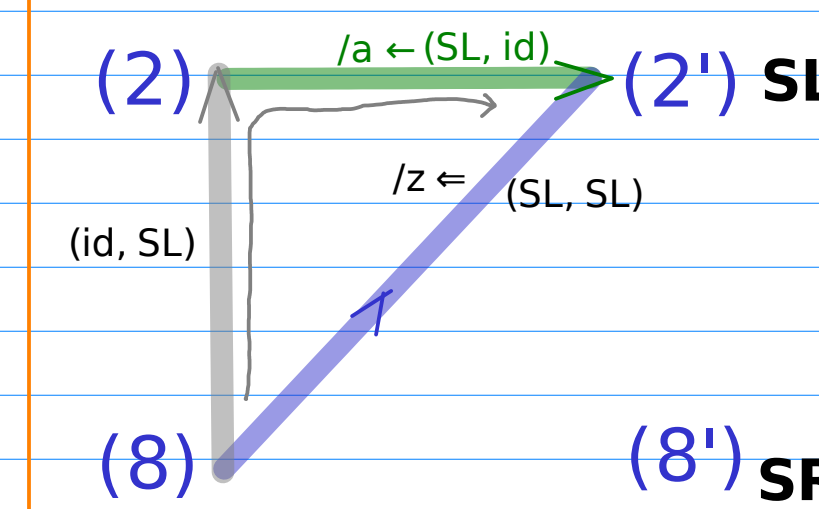
A Shifting **Shifted Sequence 1**

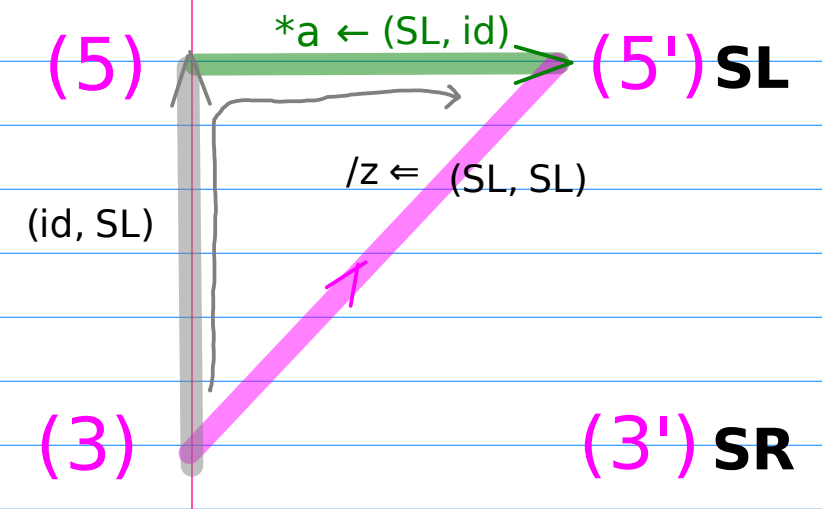
Unshifted Sequence **x** → **Shifted Sequence 1** **x'**

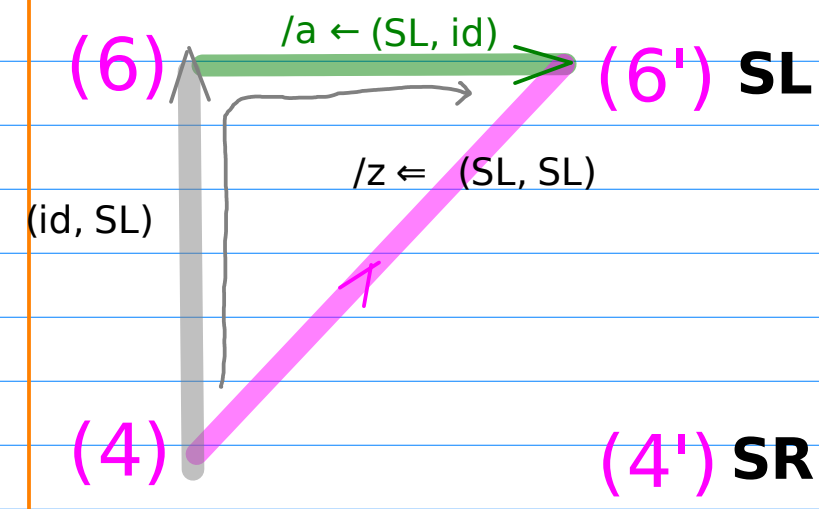
(id, SL) + (SL, id) = (SL, SL)











Decomposition of Exp and Rng Shifts (3)

A Shifting Shifted Sequence 1

Unshifted Sequence x → Shifted Sequence 1 x'

(SR, id) + (id, SR) + (SR, id) = (SR2, SR)